

## Introduction

Financial risk management, employee remuneration, strategic investment decision making and the pricing of certain funding instruments may involve the use of options (interest rate options, employee stock options, real options and bonds with embedded options, respectively). This paper discusses basic option features ('plain vanilla' rather than 'exotic' options) and pricing methods ('Binomial Option Pricing', 'Black-Scholes Model', 'Simulation').

## Option Features

In return for the payment of a 'premium' (the 'fair value' of the option), a purchaser becomes owner of an option that gives them the right (without any obligation) to buy ('Call' Option / 'CO') or sell ('Put' Option / 'PO') an asset, or lock into a rate (interest rate or currency exchange rate, for example) at a stated price or rate ('exercise' or 'strike' price) at some future specified date ('exercise date') before such a right expires ('expiry date'). The exercise date may be at any specified time before expiry ('American' option) or on expiry ('European' option).

With a stock option, a CO holder would benefit if the stock price increases above the exercise price, sufficiently to cover the premium cost: the option would be exercised, the stock purchased at the lower exercise price and immediately sold at the higher market price (profit = sale price – exercise price – option premium, ignoring taxes). A fall in the stock price could make a profit for a PO holder: the stock would be purchased at the lower market price and immediately sold at the higher exercise price (profit = exercise price – stock price – option premium). The discussion that follows will concentrate on COs.

## Pricing Approach

If a CO could only be exercised immediately after it was purchased, the premium ('P') paid to acquire the option would be its 'Intrinsic Value' / 'IV',  $S - X$  (where S is the stock price and X the exercise price), so that the total cost to acquire the share ( $P + X$ ) was the same as if purchased in the market (S). The option would be 'In-The-Money', since an immediate profit could be made (ignoring the premium). If the share price was less than the exercise price, the CO would be 'Out-Of-The-Money' and worthless.

Of course, a CO would be purchased in the hope that an unlimited 'payoff' could be achieved as the stock price increased. Some options can be exercised before the expiry ('American' option), while others can only be exercised at expiry ('European' option). At any given date during the term, the fair value of the option will equal its value if exercised immediately (IV) plus an extra value component ('Time Value') equal to the 'Present Value' / 'PV' of the expected incremental value from waiting until a later date in the hope the stock price rises:

$$\text{Option Premium (Value)} = \text{Intrinsic Value} + \text{Time Value}$$

The future payoff from delaying an exercise depends on how the stock price is expected to change over the term. If the stock price falls below the exercise price, it's IV will be zero. An option pricing model forecasts future stock prices at various dates until the expiry date, assuming some probability distribution for those prices, and ignores scenarios where the payoff is negative (when the option would not be exercised, and the loss suffered would be the premium paid).

Option pricing models, generally:

- forecast a range of stock prices at future exercise dates, assuming some probability distribution and degree of price variability (volatility);
- only consider future scenarios where the option is in the money (forecast prices that give a positive IV);
- calculate the PV of the future positive IV scenarios, weighted by the probability of their occurrence

The two basic models take different approaches to forecasting stock prices, whether 'jumping' over some 'discrete' time interval (day, week, month etc) or continuously changing.

## Pricing Relevance

The fair value of an option, as estimated by an option pricing model, is required for several purposes:

- Measurement of the grant date fair value of employee options and other share-based payment transactions recognised under IFRS 2 for financial reporting purposes (see below).
- Valuing a pool of existing options at the date a business is valued using DCF techniques, in order to allocate equity value to ordinary shareholders and option-holders (see the article 'Business Valuation – Part 1 Cash Flows & Value' and below).
- Valuing complex capital structures that have instruments with embedded options and different rights (see article 'Security Valuation – Option Pricing Method').
- Determining the value attributable to options embedded in financial instruments, such as convertible bonds and preference shares and callable bonds (see the article 'Security Valuation – Convertibles').
- Determining the value of any managerial decision-making flexibility when making capital investment decisions that involve expansion, delay or abandonment 'real' options.

## Employee Stock Options

### *Features*

Call Options may be granted to employees as part of their remuneration, with or without 'vesting' conditions that need to be fulfilled before the option 'vests'. Vesting conditions may require the employee to remain employed for a minimum length of service ('Service Conditions'). They may also have a condition that certain internal financial metrics are achieved ('Non-Market Performance Condition') or that the company share price reaches a stated level ('Market Performance Condition'). If these conditions are not met, the options are forfeited. If the options vest but are not exercised before the expiry (say 7 – 10 years), they lapse and are cancelled. Vesting can take place over time or immediately on some date ('cliff' structure).

The exercise price may be set at the fair value or market price of the underlying shares at the date they are granted ('at-the-money'), although it could be variable and depend on a measure of the stock price in the future (e.g. moving average or maximum over some period) or even zero. Exercising the option is usually the only way an employee can benefit financially from holding the option, since they cannot be sold.

### *Fair Value Estimation*

Estimating the FV of employee stock options using an option pricing model like the Binomial Method ('BM') (or 'Lattice Method') or Black-Scholes Model ('BSM') is made complicated by the following:

- The earliest date an employee can exercise the option may depend on contractual vesting conditions.
- The post-vesting date when the employee actually exercises the option will depend on when they decide to exercise, so an option model needs to estimate the expected term of the option based on an assumption about exercise behaviour. Options vested at the valuation date could be exercised immediately (if in-the-money), evenly between the valuation and expiry dates (on average at the mid-point), on expiry or according to some other method.
- Options may be grouped into 'cohorts' that share similar features, allowing valuation for the whole cohort any material difference compared to pricing each option separately.
- The stock price input for the valuation model needs to incorporate the option value per share, hence circularity is involved. This is possible by adjusting the model to allow for option value and dilution, as for a warrant model, to calculate an equity value per share that includes the option value (by dividing the combined DCF equity value + option FV by the combined shares outstanding and number of options) which is then used as the stock price input in the model.

For example, assume a company with an equity DCF value per share of £10.00 (equity DCF £100m outstanding shares 10m) has 1m options granted with an average exercise price of £8.00, using a

warrant pricing model with the inputs below produces an adjusted equity value per share of £9.09. (Based on the approach in <https://pages.stern.nyu.edu/~adamodar/pc/warrant.xls> ).

Assumptions		Diluted value				
Current share price (S)	£ 10.00	Options	£ 4.2268	1 m	9.09 %	£ 4,226,812
Shares outstanding	# 10,000,000	Non-options	£10.0000	10 m	90.91 %	£ 95,773,188
Equity Value	£ 100,000,000	Equity Value	£ 9.0909	11 m	100.00%	£100,000,000
Exercise Price (X)	£ 8.00					
Options	# 1,000,000					
Risk Free Rate (%) (continuous)(r)	5.00 %					
Volatility (%) (standard deviation)	40.00 %					
Time to Expiry (years)(T)	5.00 yrs					
Dividend yield	1.00 %					
Option valuation						
Dividends - yield (%) (q)						1.00%
Warrant adjusted share price (10m x £10.00 + 1m x £4.2268) / (10m + 1m) - circularity		£				9.48
Share price adjusted to 9.48 x EXP (1.00% x 5)		£				9.01
$d1 = (\ln(9.48 / 8.00) + (5.00\% - 1.00\% + 0.5 \times 40.00\%^2) \times 5.00) / (0.5 \times 40.00\% \times \text{SQRT}(5.00))$						0.8600
$d2 = 0.8600 - 40.00\% \times \text{SQRT}(5)$						-0.0344
$N(d1) = \text{NORMSDIST}(0.8600)$						0.8051
$N(d2) = \text{NORMSDIST}(-0.0344)$						0.4863
Share price ( £9.01 ) x N(d1)		£				7.2565
PV of ex price {£8.00 x EXP(-5.00 x 5) } x N(d2)		£				3.0297
Call Price		£				4.2268

### Recognising the Option Expense under IFRS

Under IFRS 2, the FV of the stock option is measured at the grant date (use of the BM or BSM is permitted) and is not subsequently re-valued. The option FV is expensed to profit and loss over the vesting period (with a corresponding increase in equity), depending on the proportion of options expected to vest (the allocation is intended to match the period over which the employee services are provided). If there are no vesting conditions, the option FV is immediately recognised in full.

Only Market Performance Conditions are considered when estimating the option FV (IFRS 2 para. 19), typically using the BM, as the BSM is unlikely to be able to handle them. Service and Non-Market Performance Conditions are incorporated via an adjustment to the number of options and do not affect grant date FV ('modified grant date' method).

An estimate of the proportion of options expected to vest (some employees may leave, for example so the options never vest) is made on each reporting date during the vesting period. In the first year the charge will be the vesting fair value (option FV at grant date x options granted x % expected to vest) divided by the vesting period in years. If the expected vesting proportion is constant, the annual charge will be constant. If the second-year expectation changes, the second-year charge will differ (the first year charge is not affected), so a cumulative catch up adjustment is made in that year. In the final year, an adjustment is made

to ensure the actual number of options vesting is accounted for (see examples in EY's International GAAP 2025 section 6.2.1.A, p.2589).

### *Taxation of Options (UK)*

In the UK, the granting of an option over shares would, if the qualifying conditions are met, enable the company to deduct an amount for corporation tax, but only if the option is exercised and the employee obtains beneficial ownership of the shares. Relief is given on 'an amount equal to the market value of the shares when they are acquired, less the total amount or value of any consideration given by any person in relation to the obtaining of the option or to the acquisition of the shares....' [s1018 Corporation Tax Act 2009] (although some 'restricted' and convertible securities have a different relief provision). 'The relief is given for the accounting period in which the shares are acquired...as a deduction in calculating the profits of the qualifying business for corporation tax purposes....' [s1021 Corporation Tax Act 2009] 'The statutory deduction overrides the accounting treatment' (HMRC Business Income Manual BIM44265). For non-qualifying shares, the deduction would be equal to the amount treated as employment income for the employee.

The expense charged to the income statement for the fair value of employee stock option grants (allocated over the vesting period under IFRS 2) differs to the amount deductible for tax purposes, which will arise in later periods when the options are exercised (the tax deduction being the intrinsic value at the exercise date: share price less exercise price, as discussed above). There is a mismatch between the accounting and taxable profits, which requires an adjustment via deferred tax (discussed in Part 4). The tax deduction given after the reporting date for options granted up to that date (based on estimates) is the tax base; the carrying amount is nil, so the deferred tax asset equals the tax rate x tax base (see EY International GAAP 2025 p.2491).

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## Suggested reading

### Books:

- Chriss, N.A. (1997) *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill.
- Clewlow, L. & Strickland, C. (1998) *Implementing Derivatives Models*, Wiley
- Cox, J. C., and M. Rubinstein (1985) *Option Markets*. New Jersey: Prentice Hall.
- Ernst & Young (2025) *International GAAP 2025* [https://www.ey.com/en\\_gl/technical/ifrs-technical-resources/international-gaap-2025-the-global-perspective-on-ifrs](https://www.ey.com/en_gl/technical/ifrs-technical-resources/international-gaap-2025-the-global-perspective-on-ifrs)
- Hull, J.C. (2022) *Options, Futures, and Other Derivatives* (11th Edition). Pearson
- James, P. (2003) *Option Theory*. Chichester, W.Sussex: Wiley
- Jarrow, R., and A. Rudd (1983) *Option Pricing*. Homewood, IL: Richard Irwin, Inc.
- McDonald, R.L (2003) *Derivatives Markets*. Boston: Addison Wesley.
- Rendleman Jr., R. J. (2002) *Applied Derivatives: Options, Futures, and Swaps*. Oxford: Blackwell Publishers
- Rubinstein, M. (1999) *Rubinstein on Derivatives: Futures, Options and Dynamic Strategies*. London: Risk Publications.
- Sadr, A. (2022) *Mathematical Techniques in Finance: An Introduction*. Wiley
- Tan, P., Lim, C.Y., & Kuah, E.W.(2020) *Advanced Financial Accounting: An IFRS® Standards Approach* (4<sup>th</sup> ed.) McGraw-Hill

### Papers

- Black, F., and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81(3), 1973, 637-659.
- Cox, J., S. Ross, and M. Rubinstein (1979), "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7(3), 1979, 229-264.
- Merton, R.C. (1973), "The Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*, 4(1), 1973, 141-183.

Pricing without Dividends

*Single Step*

*Binomial Model*

The most common discrete version of the models is the 'Binomial' Model ('BM'). In the BM it is assumed that, over a single time step, the current price of an asset ( $S_0$ ) can increase by a factor 'u' ( $S_{1u} = u.S_0$ ) or decrease by a factor 'd' ( $S_{1d} = d.S_0$ ) with true probabilities of  $p^*$  and  $1 - p^*$ , respectively (the sum of all probabilities of possible states at any time must equal 1).  $S_0$  will therefore be the expected price at the next step  $S_1$  (probability weighted price) discounted at the risk-adjusted rate (i.e. cost of equity). In order to value the option payoff using the risk-free rate (used in the Black-Scholes Model), an adjustment is made to the true probabilities to ensure the up and down states are 'Certainty Equivalents' / 'CE':

$$\frac{\text{Expected price} \times \text{true probability}}{1 + \text{risk adjusted rate}} = \frac{\text{CE price} \times \text{'Risk Neutral' probability}}{1 + \text{risk free rate}}$$

$$S_0 = \frac{u \cdot S_0 \cdot p^* + d \cdot S_0 (1-p^*)}{1 + R_{fd} + \text{risk premium}} = \frac{u \cdot S_0 \cdot p + d \cdot S_0 (1-p)}{1 + R_{fd}}$$

Where:

- $p^*$  true probability
- $p$  risk neutral probability (not a real probability but an adjusted true probability)
- $R_{fd}$  risk free rate (discrete rate)

Taking a simple one step / two state example, assuming the exercise price is the grant date market price ( $X = S_0 = 150.00$ ), time to expiry 1 year, volatility 40.0%, the discrete risk free rate 5.13%<sup>1</sup> and the risk premium 2.98% (in CAPM this would equal the geared beta x market equity risk premium), using the approach of Cox, Ross & Rubinstein (1979) and ignoring dividends for now, the up and down factors  $u = 1.4918$  ( $= e^{(40\% \times \sqrt{1.0})}$ ) and  $d = 0.6703$  ( $= 1 / u$ ), would give up and down state asset prices and intrinsic values of  $S_{u1} = 223.77$  and  $S_{d1} = 100.55$ . Assuming these up and down states have an equal true probability ( $p^* = 0.5$ ), we can compute the risk neutral probability as  $p^* = 0.4637$ :

$$150 = \frac{(1.4918 \times 150 \times 0.5 + 0.6703 \times 150 \times (1 - 0.5))}{1 + 5.13\% + 2.98\%} = \frac{(1.4918 \times 150 \times 0.4637 + 0.6703 \times 150 (1 - 0.4637))}{1 + 5.13\%}$$

$$150 = \frac{162.16}{1 + 8.11\%} = \frac{150.68}{1 + 5.13\%}$$

where

$$\begin{aligned}
 p^* &= \frac{(1 + R_{fd} - d)}{(u - d)} \\
 &= \frac{(1 + 5.13\% - 0.6703)}{(1.4918 - 0.6703)} \\
 &= 0.4637
 \end{aligned}$$

The expected price using risk neutral probabilities (150.68) is the certainty equivalent price.

The up and down factors depend on how volatile the asset price is expected to be over the period to expiry (measured as the standard deviation of returns). The more volatile the asset returns, the greater the likelihood the asset price will increase above the exercise price (for a CO), increasing the option FV. How this volatility is incorporated in u and d depends on the method chosen (the CRR method is used above, but there are others such as Jarrow and Rudd (1983)).

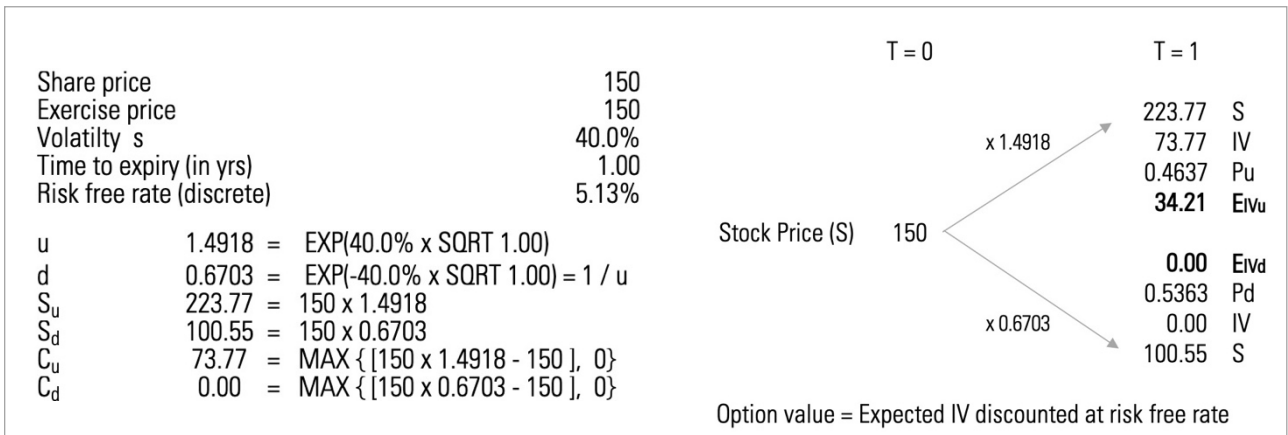
<sup>1</sup> The relationship between the discrete rate ( $R_{fd}$ ) and continuous rate ( $R_{fc}$ ) is  $R_{fd} = (e^{R_{fc}} - 1)$  and  $R_{fc} = \ln(1 + R_{fd})$  where 'e' is the exponential function (2.71828...) and 'ln' the natural logarithm. For example, a 5.0% nominal rate continuously compounded would give an effective annual rate of 5.1271%

Period	Year	Compounded	Continuou:	Effective
Annual	1	1	5.0000%	5.0000%
Semi-annual	0.5	2	5.0000%	5.0625%
Quarterly	0.25	4	5.0000%	5.0945%
Monthly	0.0833333	12	5.0000%	5.1162%
Weekly	0.0192308	52	5.0000%	5.1246%
Daily	0.0027397	365	5.0000%	5.1267%
Hourly	0.0001142	8,760	5.0000%	5.1271%
Every minute	0.0000019	525,600	5.0000%	5.1271%

$$\begin{aligned}
 (1 + 5.0000\% / 525,600)^{525,600} - 1 &= 5.1271\% \\
 \text{EXP}(5.0000\%) - 1 &= 5.1271\%
 \end{aligned}$$

The expected price using risk neutral probabilities would suggest a modest IV of 0.68 (=150.68 – 150.00), however this ignores the fact that the down state price would have zero IV as the call option would be worthless (out-of-the-money). The asset prices and IV at the up and down states using risk neutral probabilities are:  $S_{u1} = 223.77$  (=150.00 x 1.4918),  $IV_{u1} = 73.77$  (=223.77 – 150.00),  $S_{d1} = 100.55$ ,  $IV_{d1} = 0$  (= max {0, 100.55 – 150.00}). The down state price is ignored in the valuation, so that the option value is calculated as:

$$32.54 = \frac{IV_u P_u + IV_d P_d}{1 + R_{fd}} = \frac{73.77 \times 0.4637 + 0.00 \times (1 - 0.4637)}{1 + 5.13\%}$$



The same call option price can be calculated in terms of a replicating portfolio approach. If the seller ('writer') of the option uses the proceeds from the option grant (32.54 premium) and a loan (57.26 at 5.13% risk free rate) to buy a hypothetical 0.5987 fraction of a share for 89.80 (= 0.5987 x 150.00 share price at  $T_0$ ), its payoff in the two up and down states would be zero: if the option is exercised for 150.00 in the upstate, the writer would pay off the loan with interest (60.2 = 57.26 x 1.0513) and be left with 89.80, offsetting the initial share cost; if the option is not exercised in the down state, the writer would sell the stock in the market for 60.2 (= 0.5987 x 100.55) and pay off the loan. The call option value (32.54) equals the cost of the share (89.80) less the loan (57.26), or, in terms of the current share price  $S_0$

$$\text{Call option (32.54)} = S_0 \times \delta - B = 150.00 \times 0.5987 - 57.26$$

Where:

$$\delta \text{ ('Hedge Ratio')} = \frac{IV_u - IV_d}{S_0 (u - d)} = \frac{73.77 - 0.00}{150 (1.4918 - 0.6703)}$$

$$B \text{ Loan} = \frac{(d \times IV_u - u \times IV_d)}{(u - d)} \times \frac{1}{(1 + Rf)} = \frac{(0.6703 \times 73.77) - 0}{(1.4918 - 0.6703)} \times \frac{1}{1.0513}$$

### Black Scholes Model

The well-known Black-Scholes Model ('BSM') (a derivation can be found elsewhere) is the continuous time version of the BM, and attempts, like the BM, to estimate future probability weighted expected prices and positive payoffs. It assumes the option can only be exercised at expiry (European Option), unlike the BM which allows American options exercisable at any date.

The BSM is based on the replicating portfolio approach and for a call option can be shown as:

$$\text{Call option} = S_0 \times \delta - B$$

Where

$$\delta = N(d_1)$$

$$B = \text{PV of Exercise Price} \times N(d_2)$$

*Black-Scholes Model (no dividends)*

$$\text{Call option value} = S N(d_1) - X e^{-rT} N(d_2)$$

$$\text{Put option value} = X e^{-rT} N(-d_2) - S N(-d_1)$$

S Current stock price

X Exercise Price

T Time to maturity

r Risk free rate (continuously compounded)

e The base of natural logarithms, constant = 2.1728 .....

$X e^{-rT}$  The amount of cash needed to be invested over a period of time T at a continuously compounded interest rate r in order to receive X at maturity (i.e. it is the PV of the exercise price continuously discounted at the risk free rate).

$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(S/X) + rT}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T}$$

$$d_2 = \frac{\ln(S/X) + (r - 0.5\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where  $\sigma$  is the annualised standard deviation of logarithmic stock returns

N(x) The cumulative probability function for a standardised normal variable. N(d1) and N(d2) will be discussed later, in the context of the Binomial model.

The probability N(d1) is equivalent to the hedge ratio and indicates how sensitive the option price is to the underlying share price.  $S N(d1)$  represents the PV from receiving the share, conditional on the option being exercised. The probability N(d2) indicates how likely the option will be exercised, so  $X e^{-rT} N(d2)$  represents the PV of the price paid to acquire the share.

The BSM uses logarithmic returns and defines the relationship between the current price and exercise price in terms of  $\ln(S_0 / X)$  (the equivalent of  $S_0 - X$  in a normal distribution), adjusting this to estimate the forward value by the risk free rate of return.

Using the example for the BM, we can value the option assuming exercising occurs after 1 year. The call option value is 27.0344, lower than the BM because the BSM assumes continuous compounding over the 1 year whereas the BM is a discrete version with one compounding:

Current share price (S)	150.00	Time to Expiry (years)(T)	1.00 yrs
Exercise Price (X)	150.00	Risk Free Rate (%) (discrete)(r)	5.13%
Volatility (%)(standard deviation)	40.00%	Risk Free Rate (%) (continuous)(r)	5.00%
d1 = (LN(150.00 / 150.00 ) + (5.00% - 0.00% + 0.5 x 40.00%^2) x 1.00) / (0.5 x 40.00% x SQRT(1.00))			0.325000
d2 = 0.3250 - 40.00% x SQRT(1)			-0.075000
N(d1)= NORMSDIST(0.3250)			0.6274
N(d2)= NORMSDIST(-0.0750)			0.4701
Share price ( 150.00 ) x N(d1)			94.11
PV of ex price {150.00 x EXP(-5.00 x 1) } x N(d2)			67.08
Call Price			<b>27.0344</b>

The value of the call option will increase as: share price ↑ (measured by delta\*), volatility ↑(vega\*), time to expiry ↑ (theta\*), risk free rate ↑(rho\*) and the lower the fixed exercise price (\* the 'Greeks').

As the number of time periods in the BM increases, the option value converges to the BSM. To equate the BM to the BSM, the time steps in the BM need to be increased significantly, which requires the use of a multi-step tree.

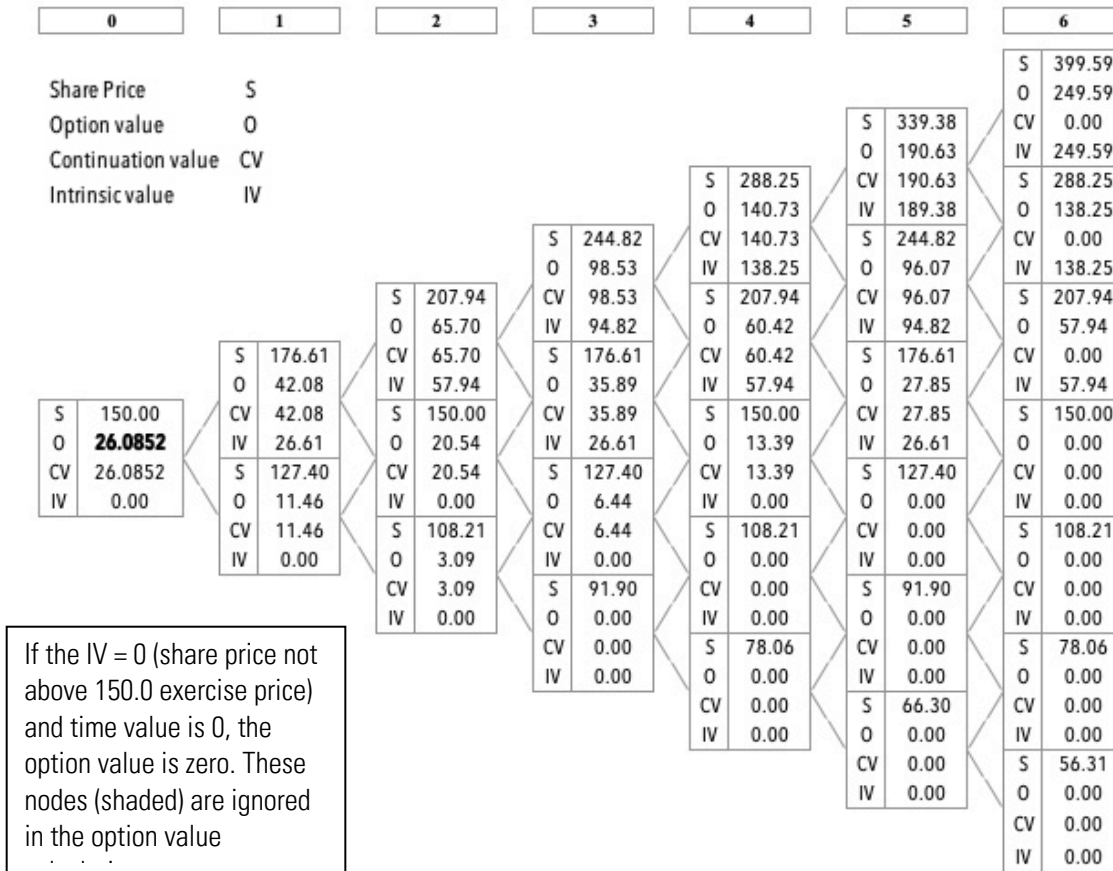
### *Multi-Step*

A 'Binomial Tree' (or 'Lattice') shows expected stock prices, Intrinsic Values and option values for each period until expiry. At the end of each stock price path (a 'node' – for example, the up and down state nodes in the above single period example), the stock price can increase or decrease at the next time period, so that nodes multiply geometrically (trinomial models add a third path between the up and down binomial states). If the option can be exercised at any date before expiry (American option), the value at each node will be the Intrinsic Value at that node or, if higher because of time value, the present value of the option at the next period (the 'Continuing Value'):

$$\text{Continuing Value}_t = \frac{p \times (\text{Option Value}_{t+1u}) + (1 - p) \times (\text{Option Value}_{t+1d})}{1 + \text{risk free rate } (R_{fd})}$$

At the final time period  $t_n$  (the expiry), the option value will simply be  $\max \{0, S_n - X\}$  i.e. the IV or zero as for the one period example above.

For a European option, the value at any time-period before expiry will be the Continuing Value, since it cannot be exercised until expiry. Using the same information, but assuming the 1-year period is divided into 6 time steps, the following tree and an option value (26.0852) is shown:



$u = 1.1774$        $U = \text{EXP}(40.0\% \times \text{SQRT}(1/6))$   
 $d = 0.8493$        $D = 1/1.1774$   
 Risk-free return       $1.0084$        $r = \text{EXP}(5.00\% \times 0.1667)$   
 Risk-neutral probability       $0.4848$        $p = (1.0084 - 0.8493)/(1.1774 - 0.8493)$

*Example of workings - cell U<sup>4</sup>*  
 $288.25 = 244.82 \times 1.1774$   
 $140.73 = \text{MAX}\{140.73, 138.25\}$   
 $140.73 = [0.4848 \times 190.63 + (1-0.4848) \times 96.07] / 1.0084$   
 $138.25 = 288.25 - 150.00$

A European Call Option on a non-dividend paying stock will have the same value as its American counterpart, since it would never be optimal to exercise the latter early (this is not the case if dividends are paid).

### BM-BSM Convergence

The call option value calculated above (26.0852) can be calculated using the Binomial distribution, a discrete probability distribution which measures the number of successes (probability 'p') and failures (probability '1-p') in a sequence of independent trials. The call option can be valued as:

$$C_0 = S_0 \times \sum_{n=a}^T \left\{ A \times \left( \frac{u^n d^{T-n}}{(1+r)^T} \right) \right\} - \frac{X}{(1+r)^T} \times \sum_{n=a}^T \left\{ A \right\}$$

PV of expected stock price at each node, where exercising the option is optimal (probability weighted).

PV of exercise price at each node where exercising is optimal (probability weighted).

Where:

$$A = \frac{T!}{n!(T-n)!} p^n (1-p)^{T-n} = \text{possible combinations} \times \text{probability of success}$$

Each node can be reached via  $T! / n!(T-n)!$  possible paths.

Node 'n'	p	p <sup>n</sup>	(1-p) <sup>T-n</sup>	P <sub>u</sub> x P <sub>d</sub>	Paths	Binomial probability	In-the-money?	Prob of exercise	State Price	Yr 6 price (S <sub>T</sub> )	PV of Expected value of S <sub>T</sub> / S <sub>0</sub>
A	B	C = A x B	D	E = C x D	F	G = E x F		A	A / e <sup>r</sup>	u <sup>n</sup> x d <sup>(T-n)</sup>	A x B / e <sup>r</sup>
6	0.4848	0.0130	1.0000	0.0130	1	0.0130	✓	0.0130	0.0123	2.6639	0.0329
5	0.4848	0.0268	0.5152	0.0138	6	0.0828	✓	0.0828	0.0787	1.9217	0.1513
4	0.4848	0.0552	0.2655	0.0147	15	0.2199	✓	0.2199	0.2092	1.3862	0.2900
3	0.4848	0.1139	0.1368	0.0156	20	0.3116	✗				
2	0.4848	0.2350	0.0705	0.0166	15	0.2484	✗				
1	0.4848	0.4848	0.0363	0.0176	6	0.1056	✗				
0	0.4848	1.0000	0.0187	0.0187	1	0.0187	✗				
						<b>1.0000</b>		<b>0.3157</b>			<b>0.4742</b>

Exercise Price	150.00	Share Price	150.00
Discount factor	x 0.9512	Payoff factor	0.4742
Probability of exercise	x 0.3157		
	45.0389		71.1240

Value **26.0852**

$$\text{Node 4} = (0.4848^4) \times \{(1 - 0.4848)^{(5-4)}\} = 0.0147$$

Node 'n'	Share price S <sub>i</sub>	Return (S <sub>i</sub> /S <sub>0</sub> ) - 1	Return ln(S <sub>i</sub> /S <sub>0</sub> )
6	399.59	166.4%	98.0%
5	288.25	92.2%	65.3%
4	207.94	38.6%	32.7%
3	150.00	0.0%	0.0%
2	108.21	-27.9%	-32.7%
1	78.06	-48.0%	-65.3%
0	56.31	-62.5%	-98.0%

	Discrete	Continuous
Mean μ	157.69	5.13%
Variance σ <sup>2</sup>	4214.13	18.7%
σ	64.92	43.3%
r	-0.5v	-3.000%

The Excel function can also be used:

Up factor	1.1774	
Down factor	0.8493	
Min up moves before in-the-money (integer)	3.0000	= INT[ LN [ 150 [X] / 150 [So] x 0.8493^6 ] / LN [ 1.1774 / 0.8493 ] ]
Total trials	6	Time steps
Risk neutral probability	0.4848	
Risk free return per time step	1.0084	r = EXP(5%*0.1667)
Probability of success	0.5660	= 0.4848 x ( 1.1774 / 1.0084 )
Current share price factor	0.4742	= 1 - BINOMDIST(3, 6, 0.5660, TRUE)
Exercise price factor	0.3157	= 1 - BINOMDIST(3, 6, 0.4848, TRUE)
Call Price	<b>26.0852</b>	= 150 x 0.4742 - 150 / 1.0084^(6) x 0.3157

TRUE' is a signal to Excel that the distribution must be cumulative (i.e. probability higher than X)

The number of nodes can be increased to check convergence to the BSM, here up to 1000 time steps:

EUROPEAN CALL OPTION - BINOMIAL - 1, 2, 3, ..., 1000 TIME PERIODS - 1 YEAR (EXTRACTS)									
Time Steps in 1 year	Up	Down	1 + r	Risk neutral p	Min up moves	P* of success	P. P(x)	P	Binomial Call
1	1.491825	0.670320	1.0513	0.463724	0	0.658055	0.6581	0.4637	<b>32.5421</b>
2	1.326896	0.753638	1.0253	0.473917	1	0.613313	0.3762	0.2246	24.3763
3	1.259784	0.793787	1.0168	0.478586	1	0.592949	0.6378	0.4679	28.9109
4	1.221403	0.818731	1.0126	0.481403	2	0.580683	0.4421	0.2851	25.6323
5	1.195884	0.836202	1.0101	0.483339	2	0.572266	0.6336	0.4688	28.1554
6	1.177389	0.849337	1.0084	0.484774	3	0.566031	0.4742	0.3157	<b>26.0852</b>
7	1.163213	0.859688	1.0072	0.485893	3	0.561174	0.6318	0.4692	27.8324
8	1.151910	0.868123	1.0063	0.486796	4	0.557251	0.4939	0.3348	26.3174
9	1.142631	0.875173	1.0056	0.487546	4	0.553998	0.6308	0.4694	27.6536
996	1.012755	0.987405	1.0001	0.498812	497	0.505149	0.6394	0.4827	27.0286
997	1.012749	0.987412	1.0001	0.498812	498	0.505146	0.6274	0.4701	27.0400
998	1.012742	0.987418	1.0001	0.498813	499	0.505144	0.6154	0.4575	27.0286
999	1.012736	0.987424	1.0001	0.498814	499	0.505141	0.6274	0.4701	27.0399
1000	1.012729	0.987431	1.0001	0.498814	500	0.505139	0.6154	0.4575	<b>27.0286</b>

\* Workings - time step 999:

Up	= EXP(40.00% x SQRT(1 / 999))	= 1.012736	Down = 1 / U
1 + r*	= EXP(5.00% x SQRT(1 / 999))	= 1.0001	
p*	= (1.0001 - 0.9874) / (1.0127 - 0.9874)	= 0.498814	
Min moves	= INT(LN(150.0 / (150.0 x (0.9874^999))) / LN(1.0127 / 0.9874))	= 499	
Pr success	= 0.4988 x 1.0127 / 1.0001	= 0.505141	
(P)(Px)	= (1 - BINOMDIST(499, 999, 0.5051, TRUE))	= 0.6274	
P	= (1 - BINOMDIST(499, 999, 0.4988, TRUE))	= 0.4701	
Binom Call	= 0.6274 x 150.0 - 0.4701 x 142.68 (the PV of the exercise price)	= 27.04	

## Pricing with Dividends

Dividends reduce the value of a call option since the option holder does not receive the cash payout, which is perceived to reduce equity value marginally. One method of incorporating dividends into the BM is by assuming they are paid at a continuous yield ( $q\%$ ) and adjusting downwards the risk neutral probability to  $P = (e^{(r-q)t} - d) / (u - d)$ . Using the 6 period example, assuming a 6.00% annual dividend yield (1.00% each time step), the option value reduces from 26.0852 to 22.1798 for a European option and 22.3702 for an American option. In the latter case, it is optimal to exercise early (the layout has been rotated 45 degrees for presentation purposes).

European							American								
	0	1	2	3	4	5	6		0	1	2	3	4	5	6
S	150.00	176.61	207.94	244.82	288.25	339.38	399.59	S	150.00	176.61	207.94	244.82	288.25	339.38	399.59
IV	0.00	0.00	0.00	0.00	0.00	0.00	249.59	IV	0.00	26.61	57.94	94.82	138.25	189.38	249.59
TV	22.18	37.36	60.67	94.07	137.75	189.13	0.00	TV	22.37	11.07	3.23	0.00	0.00	0.00	0.00
O	<b>22.1798</b>	<b>37.36</b>	<b>60.67</b>	<b>94.07</b>	<b>137.75</b>	<b>189.13</b>	<b>249.59</b>	O	<b>22.3702</b>	<b>37.68</b>	<b>61.17</b>	<b>94.82</b>	<b>138.25</b>	<b>189.38</b>	<b>249.59</b>
	S	127.40	150.00	176.61	207.94	244.82	288.25		S	127.40	150.00	176.61	207.94	244.82	288.25
	IV	0.00	0.00	0.00	0.00	0.00	138.25		IV	0.00	0.00	26.61	57.94	94.82	138.25
	TV	9.48	17.85	32.69	57.44	94.57	0.00		TV	9.57	18.01	6.37	0.00	0.00	0.00
	O	<b>9.48</b>	<b>17.85</b>	<b>32.69</b>	<b>57.44</b>	<b>94.57</b>	<b>138.25</b>		O	<b>9.57</b>	<b>18.01</b>	<b>32.98</b>	<b>57.94</b>	<b>94.82</b>	<b>138.25</b>
	S	108.21	127.40	150.00	176.61	207.94			S	108.21	127.40	150.00	176.61	207.94	
	IV	0.00	0.00	0.00	0.00	57.94			IV	0.00	0.00	0.00	0.00	26.61	57.94
	TV	2.48	5.46	11.99	26.36	0.00			TV	2.51	5.51	12.11	0.00	0.00	0.00
	O	<b>2.48</b>	<b>5.46</b>	<b>11.99</b>	<b>26.36</b>	<b>57.94</b>			O	<b>2.51</b>	<b>5.51</b>	<b>12.11</b>	<b>26.61</b>	<b>57.94</b>	
	S	91.90	108.21	127.40	150.00				S	91.90	108.21	127.40	150.00		
	IV	0.00	0.00	0.00	0.00				IV	0.00	0.00	0.00	0.00	0.00	0.00
	TV	0.00	0.00	0.00	0.00				TV	0.00	0.00	0.00	0.00	0.00	0.00
	O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>				O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
S	Share Price		S	78.06	91.90	108.21			S	78.06	91.90	108.21			
IV	Intrinsic Value		IV	0.00	0.00	0.00			IV	0.00	0.00	0.00	0.00	0.00	0.00
TV	Time Value		TV	0.00	0.00	0.00			TV	0.00	0.00	0.00	0.00	0.00	0.00
O	Option Value		O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>			O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
	S	66.30	78.06						S	66.30	78.06				
	IV	0.00	0.00						IV	0.00	0.00				
	TV	0.00	0.00						TV	0.00	0.00				
	O	<b>0.00</b>	<b>0.00</b>						O	<b>0.00</b>	<b>0.00</b>				
	S	56.31							S	56.31					
	IV	0.00							IV	0.00					
	TV	0.00							TV	0.00					
	O	<b>0.00</b>							O	<b>0.00</b>					

Model assumptions	
Share price today	150.00
ExercisePrice	150.00
Risk Free interest rate	5.00%
Volatility	40.0%
Expiry date	1.00 yrs
Time step (fraction of year)	1/6
Call (c) or Put (p)	c
Dividend yield per year	6.0%
Dividend yield per period	1.00%

Parameters	
u	1.1774 = EXP(40.0%*SQRT( 1 / 6))
d	0.8493 = 1/1.1774
r	0.9983 = EXP(5.00% - 1.00%) x (1 / 6)
p	0.4542 = (0.9983 - 0.8493)/(1.1774 - 0.8493)

A European option cannot be exercised before expiry, so IV will be zero until the final node
--

The BSM incorporates a continuous dividend yield by reducing the risk free rate:

Current share price (S)	150.00	Time to Expiry (years)(T)	1.00 yrs
Exercise Price (X)	150.00	Risk Free Rate (%) (discrete)(r)	5.13%
Volatility (%) (standard deviation)	40.00%	Risk Free Rate (%) (continuous)(r)	5.00%
Continuous dividends - yield (%) (q)			<b>6.00%</b>
Share Price			150.00
Share price adjusted to 150.00 x EXP (6.00% x 1)			141.26
d1 = (LN(150.00 / 150.00 ) + (5.00% - <b>6.00%</b> + 0.5 x 40.00%^2) x 1.00) / (0.5 x 40.00% x SQRT(1.00))			0.175000
d2 = 0.1750 - 40.00% x SQRT(1)			-0.225000
N(d1)= NORMSDIST(0.1750)			0.5695
N(d2)= NORMSDIST(-0.2250)			0.4110
Share price ( 141.26 ) x N(d1)			80.44
PV of ex price {150.00 x EXP(-5.00 x 1) } x N(d2)			58.64
Call Price			<b>21.8028</b>

## Monte Carlo Simulation

The BSM assumes stock prices follow a path determined by a 'Stochastic' process that involves a random element ('random walk').

$$S_t = S_0 e^{[(r - 0.5\sigma^2)t + \sigma\sqrt{t}Z]}$$

Where:

$S_t, S_0$	Share price at time t and 0
r	Risk free rate (continuous)
$\sigma$	Volatility
t	Time to expiry
Z	Random variable that follows a normal distribution (= NORMSINV(RAND())) using Excel

The share price equation can be run over as many simulated price paths as required, and for each predicted stock price the payoff can be calculated as for a European Option in the Binomial Tree (=MAX{share price – exercise price, 0}). The average payoff discounted at the risk free rate gives the Call price.