

## Introduction

This is the second part of the section on the Enterprise - Equity Value 'Bridge'. Part I discussed non-operating assets and some debt and debt-equivalent items; this Part focuses on debt and equity equivalent items that require the use of option pricing techniques to estimate their fair value, namely convertible bonds and employee stock options (Appendix 1 discusses option pricing models, parts of which can be used to value convertible bonds – Appendix 2 gives an example of a convertible bond pricing model).

## Debt and Debt-Equivalents

### *Convertible Debt*

#### *Introduction*

A convertible bond or note is a debt instrument that can be converted at the investor's option into shares of the issuing company, subject to agreed terms and conditions. On conversion, investors effectively pay an exercise price by surrendering the bonds in exchange for a stated number of shares for each bond ('Conversion Ratio'). The value received from converting ('Conversion Value' or 'Parity' = Conversion Ratio x share price) should be greater than that received from any alternative strategy (i.e. the value of the bond if held to maturity and not converted, its 'Investment Value'). The possibility of a 'payoff' means convertible investors will accept a lower coupon (or even a zero coupon) compared to an otherwise identical non-convertible debt.

#### *Early Redemption*

Most issuers will have the right to service notice to redeem ('Call') the bonds early at pre-agreed dates and prices, usually after some time has elapsed ('Non-Call Period') and at a price that preserves the economic benefit for the holder (the Investment Value). For the investor, early redemption means the potential upside gain on conversion is lost and redemption proceeds may have to be reinvested at a lower yield. The fair price of a callable bond will, therefore, be less than the fair price of an otherwise identical noncallable bond (similar coupon, maturity and risk), due to this extra risk (the difference being the value of the issuer's call option).

For the issuer, early redemption allows an issuer to refinance bonds at a lower cost, following a fall in market yields. The bonds are unlikely to be called if the call price exceeds the bond trading price (otherwise it would be cheaper to repurchase them on the market), unless there are clear economic benefits from refinancing the old bonds at that price (on an after-tax NPV basis, net of all repurchase costs). For a Convertible, the call provision can be conditional on certain events occurring ('Soft Call'),

such as the underlying share price reaching specified levels, or unconditional ('Hard Call'). The serving of a notice to call a bond should force investors to convert if the call price is less than the Conversion Value ('Forced Conversion'), so that they receive a higher amount, although any accrued interest on the bond would be foregone on conversion. Forcing conversion allows the issuer to avoid a cash payout on redemption, and allows the Convertible to be seen as a form of deferred equity financing (but with less dilution than a straight upfront issue of shares due to the lower number of shares being issued, assuming share prices have risen).

### *Valuation*

The fair price of a convertible bond can be viewed as its value as a straight bond without any conversion feature (Investment Value) plus the value of the embedded option to convert to equity, except at maturity the value will be either its equity value (when a high share price means the Conversion Value exceeds the Investment Value) or its Investment Value (the opposite at a lower share price).

At any date before maturity, it may be optimal to delay conversion due to the Time Value of the conversion option (see Appendix 1), in which case the Convertible fair price would exceed the Conversion Value and would reflect the 'Continuing Value' of the Convertible. When the Conversion Value is much greater than the Investment Value, the Convertible fair price will reflect the value of the underlying equity and its volatility, and the bond's value as straight debt will be less relevant (i.e. the impact of changes in market yields and interest rates will be less); conversely, when the Conversion Value is less than the Investment Value, the Convertible fair price will equal the Investment Value (the fair price should never fall below its value as straight debt).

The option embedded nature of a convertible means it can be valued using an option pricing model, such as the Binomial Model or Black-Scholes Model (see Appendix 1). A simple example is given in Appendix 2, where a Binomial Tree is used to estimate the Convertible fair value and its debt and equity components.

### *IFRS Accounting*

Under IAS 32, the issuer of a convertible bond (a 'Compound Financial Instrument') is required to separate the convertible fair value on initial recognition into a liability component (the PV of debt cashflows without any conversion feature - 'host contract') and a residual equity component (the conversion option, being the fair value of the convertible less the value as straight debt). If there are other embedded features, such as a call option or early redemption right, these must be separated out as well. Under IFRS 9, an investor in a convertible (a 'hybrid') is not required to separate the two components, and can recognize the convertible at fair value if certain conditions are met.

In its balance sheet, the issuer must recognise the debt component at amortised cost (discussed in Part 4) and the equity component relating to the conversion feature as equity (and not subsequently remeasure it), but only if it meets the definition of equity. If treated as an equity derivative, the 'fixed-for-fixed'

criterion would need to be met for equity classification, otherwise it is treated as an embedded derivative, and, like non-equity derivatives such as a call option, would be included as part of the liability if 'closely' related to the host contract under IFRS 9 – a call option to redeem the convertible at par or approximately amortised cost would be closely related. Conversion is not anticipated until it occurs, when the carrying amount of the debt component is transferred to equity (the consideration given by the convertible holder for the shares received on conversion is the present value of future cash flows on the convertible that the issuer is no longer required to make). Finally, convertibles will affect the diluted Earnings Per Shares as calculated under IAS 33. (See EY International GAAP (2025) p.3,535, p.3,540, and p.2,888 – 2,894 for further discussion on these issues).

### *UK Taxation*

In general, the tax treatment will follow the accounting treatment. The amount recognised by the issuer as equity under IAS 32 and IFRS 9 has no tax effect (there is one exception – see HMRC Corporate Finance Manual CFM55510). If the call option is not treated as part of the host contract it will be taxed separately as a derivative (the straight bond component will be taxed under the Loan Relationship Rules discussed in Part 4, subject to the Corporate Interest Restriction rules for the deductible amortisation charge).

## **Equity-Equivalents**

### *Employee Stock Options*

#### *Valuation*

A company which has granted employee stock options (ESOs) that remain unexercised at the valuation date has created a future claim (or 'contingent' claim) over the equity value, triggered when the options are exercised (assuming exercised at a price below market price). The cost for the company of repurchasing stock to give as options (number of options x share price) less the proceeds received on exercise (number of options x exercise price), reflects the loss of value (the Treasury method assumes exercise proceeds are used to repurchase shares at the share price with the excess shares required to top up to the number of options being new diluting shares). Similarly, if options are granted after the valuation date, a further claim on equity will arise via a reduction in expected future cash flows to which existing equity investors are entitled.

One approach is as follows:

- Options granted before the valuation date (deduct from value) - The Fair Value ('FV') of options outstanding at the valuation date can be estimated using an option pricing model and deducted from the equity DCF value. The equity value per share should be based on outstanding shares (issued shares less treasury shares) and not increased to reflect dilution arising on option exercise (the effect is already

taken into account by reducing equity value). See Appendix 1 for a discussion about option pricing models.

- Options granted after the valuation date (deduct from cash flows) - Rather than attempt to forecast future option grants and exercise behaviour, the FV of option grants can be based on a percentage applied to revenues (Damodaran (2005)) or a growth rate applied to the previous period amount (Li and Wong (2004)). This would then be deducted from earnings and cash flows (despite being a non-cash item). Deducting from Free Cash Flows estimated cash outflows arising from the granting of options over the forecast period would require a forecast of DCF share prices (equity DCF value per share at each period), exercise prices (which could be the grant date DCF share price), option grants and exercise behaviour, making the calculation tricky (see Barenbaum & Schubert (2019)). Using the BSM to measure the FV charge each period would need more assumptions for the risk free rate, volatility and dividend yield, adding more complexity.

Estimating the FV of employee stock options using an option pricing model like the Binomial Method ('BM') (or Lattice Method) or Black-Scholes Model ('BSM') is made complicated by the following:

- The earliest date an employee can exercise the option may depend on certain 'vesting' conditions.
- Options may not be exercised immediately on vesting, so an option model needs to estimate the expected term of the option based on an assumption about exercise behaviour. Options vested at the valuation date could be exercised immediately (if in-the-money), evenly between the valuation and expiry dates (on average at the mid-point), on expiry or according to some other method.
- The stock price input for the valuation model needs to incorporate the option value per share, hence circularity is involved. This is possible by adjusting the model to allow for option value and dilution, as for a warrant model, to calculate an equity value per share that includes the option value (by dividing the combined DCF equity value + option FV by the combined shares outstanding and number of options) which is then used as the stock price input in the model.

For example, assume a company with an equity DCF value per share of £10.00 (equity DCF £100m outstanding shares 10m) has 1m options granted with an average exercise price of £8.00, using a warrant pricing model with the inputs below produces an adjusted equity value per share of £9.09. (Based on the approach in <https://pages.stern.nyu.edu/~adamodar/pc/warrant.xls>).

Assumptions		Diluted value				
Current share price (S)	£ 10.00	Options	£ 4.2268	1 m	9.09 %	£ 4,226,812
Shares outstanding	# 10,000,000	Non-options	£10.0000	10 m	90.91 %	£ 95,773,188
Equity Value	£ 100,000,000	Equity Value	£ 9.0909	11 m	100.00%	£100,000,000
Exercise Price (X)	£ 8.00					
Options	# 1,000,000					
Risk Free Rate (%) (continuous)(r)	5.00 %					
Volatility (%) (standard deviation)	40.00 %					
Time to Expiry (years)(T)	5.00 yrs					
Dividend yield	1.00 %					
Option valuation						
Dividends - yield (%) (q)						1.00%
Warrant adjusted share price (10m x £10.00 + 1m x £4.2268) / (10m + 1m) - circularity		£				9.48
Share price adjusted to 9.48 x EXP (1.00% x 5)		£				9.01
$d1 = (\ln(9.48 / 8.00) + (5.00\% - 1.00\% + 0.5 \times 40.00\%^2) \times 5.00) / (0.5 \times 40.00\% \times \text{SQRT}(5.00))$						0.8600
$d2 = 0.8600 - 40.00\% \times \text{SQRT}(5)$						-0.0344
$N(d1) = \text{NORMSDIST}(0.8600)$						0.8051
$N(d2) = \text{NORMSDIST}(-0.0344)$						0.4863
Share price ( £9.01 ) x N(d1)		£				7.2565
PV of ex price { £8.00 x EXP(-5.00 x 5) } x N(d2)		£				3.0297
Call Price		£				4.2268

### IFRS Accounting

Under IFRS 2, the FV of the stock option is measured at the grant date (use of the BM or BSM is permitted) and is not subsequently re-valued. If the employee is only entitled to the option on a date ('vesting date') after a certain period ('vesting period') has elapsed, during which 'vesting conditions' have been satisfied, the option FV is expensed to profit and loss over the vesting period (with a corresponding increase in equity) depending on the proportion of options granted expected to vest (the allocation is intended to match the period over which the employee services are provided). If there are no vesting conditions, the option FV is immediately recognised in full.

Vesting conditions usually involving a minimum length of service at the company ('Service Conditions') but may also involve a condition that certain internal financial metrics are achieved ('Non-Market Performance Condition') or that the company share price reaches a stated level ('Market Performance Condition'). If these conditions are not met, the options are forfeited. Only Market Performance Conditions are taken into account when estimating the option FV (IFRS 2 para. 19), typically using the BM, as the BSM is unlikely to be able to handle them. Service and Non-Market Performance Conditions are incorporated via an adjustment to the number of options and do not affect grant date FV ('modified grant date' method).

An estimate of the proportion of options expected to vest (some employees may leave, for example so the options never vest) is made on each reporting date during the vesting period. In the first year the charge will be the vesting fair value (option FV at grant date x options granted x % expected to vest) divided by the vesting period in years. If the expected vesting proportion is constant, the annual charge will be constant. If the second year expectation changes, the second year charge will differ (the first year charge is not

affected), so a cumulative catch up adjustment is made in that year. In the final year, an adjustment is made to ensure the actual number of options vesting is accounted for (see examples in EY's International GAAP 2025 section 6.2.1.A, p.2589).

### *Taxation (UK)*

In the UK, the granting of an option over shares would, if the qualifying conditions are met, enable the company to deduct an amount for corporation tax, but only if the option is exercised and the employee obtains beneficial ownership of the shares. Relief is given on 'an amount equal to the market value of the shares when they are acquired, less the total amount or value of any consideration given by any person in relation to the obtaining of the option or to the acquisition of the shares....' [s1018 Corporation Tax Act 2009] (although some 'restricted' and convertible securities have a different relief provision). 'The relief is given for the accounting period in which the shares are acquired...as a deduction in calculating the profits of the qualifying business for corporation tax purposes....' [s1021 Corporation Tax Act 2009] 'The statutory deduction overrides the accounting treatment.' (HMRC Business Income Manual BIM44265) For non-qualifying shares, the deduction would be equal to the amount treated as employment income for the employee.

The expense charged to the income statement for the fair value of employee stock option grants (allocated over the vesting period under IFRS 2) differs to the amount deductible for tax purposes, which will arise in later periods when the options are exercised (the tax deduction being the intrinsic value at the exercise date: share price less exercise price, as discussed above). There is a mismatch between the accounting and taxable profits, which requires an adjustment via deferred tax (discussed in Part 4). The tax deduction given after the reporting date for options granted up to that date (based on estimates) is the tax base; the carrying amount is nil, so the deferred tax asset equals the tax rate x tax base (see EY International GAAP 2025 p.2491).

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## Suggested reading

### Books:

- Calamos, J. (1998) *Convertible Securities*. McGraw-Hill.
- Choudry, M., Moskovic, D., Wong, M. & Zhuoshi, S.B. (2014) *Fixed Income Markets: Management, Trading, Hedging* (2<sup>nd</sup> ed.) Wiley
- Chriss, N.A. (1997) *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill.
- Clewlow, L. & Strickland, C. (1998) *Implementing Derivatives Models*, Wiley
- Cox, J. C., and M. Rubinstein (1985) *Option Markets*. New Jersey: Prentice Hall.
- Damodaran, A. (2025) *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* (4<sup>th</sup> ed.) Wiley
- Ernst & Young (2025) *International GAAP 2025* [https://www.ey.com/en\\_gl/technical/ifrs-technical-resources/international-gaap-2025-the-global-perspective-on-ifrs](https://www.ey.com/en_gl/technical/ifrs-technical-resources/international-gaap-2025-the-global-perspective-on-ifrs)
- Fabozzi, F. (2021) *The Handbook of Fixed Income Securities* (9<sup>th</sup> ed.). McGraw-Hill.
- Holthausen, Robert.W & Zmijewski, Mark.E. (2020) *Corporate Valuation*. (2<sup>nd</sup> ed.). Cambridge.
- Hull, J.C. (2022) *Options, Futures, and Other Derivatives* (11th Edition). Pearson
- James, P. (2003) *Option Theory*. Chichester, W.Sussex: Wiley
- Jarrow, R., and A. Rudd (1983) *Option Pricing*. Homewood, IL: Richard Irwin, Inc.
- Koller, T., Goedhart, D., Wesells, D., McKinsey & Co. (2025) *Valuation: Measuring and Managing the Value of Companies* (8<sup>th</sup> ed.). Wiley
- McDonald, R.L (2003) *Derivatives Markets*. Boston: Addison Wesley.
- Phillips, G.A. (1997) *Convertible Bond Markets*. London: Macmillan Press.
- Rendleman Jr., R. J. (2002) *Applied Derivatives: Options, Futures, and Swaps*. Oxford: Blackwell Publishers
- Rubinstein, M. (1999) *Rubinstein on Derivatives: Futures, Options and Dynamic Strategies*. London: Risk Publications.
- Sadr, A. (2022) *Mathematical Techniques in Finance: An Introduction*. Wiley
- Stafford Johnson, R. (2004) *Bond Evaluation, Selection, and Management*. Oxford: Blackwell Publishing
- Tan, P., Lim, C.Y., & Kuah, E.W. (2020) *Advanced Financial Accounting: An IFRS® Standards Approach* (4<sup>th</sup> ed.) McGraw-Hill
- Tuckman, B. (2022) *Fixed Income Securities: Tools for Today's Markets*. (4<sup>th</sup> ed.) Wiley
- Woodson, H. (2002) *Global Convertible Investing*. New York: Wiley

### Papers

- Barenbaum, L. & Schubert, W. (2019) "Share-based Compensation and Firm Value," *Journal of Accounting and Finance* Vol. 19(9) 2019
- Black, F., and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81(3), 1973, 637-659.
- Cox, J., S. Ross, and M. Rubinstein (1979), "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7(3), 1979, 229-264.
- Damodaran, A (2005) "Employee Stock Options (ESOPs) and Restricted Stock: Valuation Effects and Consequences" <https://pages.stern.nyu.edu/~adamodar/pdffiles/papers/esops.pdf>
- Galai, D., and M. Schneller (1978), "Pricing Warrants and the Value of the Firm", *Journal of Finance*, 33, 1978, 1339-42.
- Goldman Sachs (1993), "Valuing convertible bonds as derivatives", Quantitative Strategies Research Notes
- Hull, J. & White, A. (2002) 'How to value employee stock options' <https://www-2.rotman.utoronto.ca/~hull/downloadablepublications/esoppaper.pdf>
- Latham & Watkins (2024) "Demystifying Modern Convertible Notes" (April 2024) <https://www.lw.com/admin/upload/SiteAttachments/Demystifying-Convertible-Bonds-April-2024.pdf>
- Li, F. & Wong, M.H.F. (2004) Employee Stock Options, Equity Valuation, and the Valuation of Option Grants using a Warrant-Pricing model" <https://utoronto.scholaris.ca/items/d5e39221-1b95-488c-8d13-f651827c9b1b>
- Mayer Brown (2025) "Convertible Bonds: An Issuer's Guide (2025)" <https://www.mayerbrown.com/en/insights/publications/2025/04/convertible-bonds-an-issuers-guide-2025>
- Merton, R.C. (1973), "The Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*, 4(1), 1973, 141-183.
- Schueler, A (2021) "Executive Compensation and Company Valuation," *Abacus*, ISSN 1467-6281 <https://www.econstor.eu/bitstream/10419/230255/1/abac.12199.pdf>
- Tsiveriotis, K., and C. Fernandes (1998), "Valuing Convertible Bonds with Credit Risk", *Journal of Fixed Income*, 8(2), 1998, 95-102.
- Wever, J.O., Smid, P.P.M. & Koning, R.H. (2003) "Pricing of convertible bonds with hard call features" <https://research.rug.nl/en/publications/pricing-of-convertible-bonds-with-hard-call-features-3>

### Introduction

An option holder has the right (without any obligation) to buy ('Call' Option / 'CO') or sell ('Put' Option / 'PO') an asset at a certain price ('exercise' or 'strike' price) at some future specified date ('exercise date') before such a right expires ('expiry date'). The exercise date may be at any specified time before expiry ('American' option) or on expiry ('European' option). The price paid to acquire the option ('option premium') is the option's Fair Value ('FV').

The exercise price of an employee stock option (a CO) given or 'granted' by the employer would normally be set equal to the market price of the underlying shares at the grant date ( $X = S_{\text{grant}}$ ). The exercise date might depend on certain 'vesting' conditions being met, and once vested the employee (option holder) would hope that  $S$  exceeds  $X$  (when the CO is 'in-the-money' with a value equal to its 'Intrinsic Value' (IV) or  $S_{\text{exercise}} - X$ ). If at time  $t_1$  the share price  $S_{t_1}$  is expected to increase by a later date  $t_2$ , then the potential profit might be greater, measured in present value terms, if the option holder delayed exercising until that future date ( $(S_{t_2} - X) / (1 + r)^{(t_2 - t_1)} > S_{t_1} - X$ ). The extra value at  $t_1$  from delaying an exercise until  $t_2$  represents the 'Time Value' ('tv'), which depends on the probability of  $S$  achieving  $S_{t_2}$  by  $t_2$  and the discount rate  $r$ .

An option pricing model forecasts future asset prices at various dates until the expiry date, assuming some probability distribution for those prices. The dates can be continuous ('Black-Scholes Model' BSM) or discrete ('Binomial Model' BM or 'Lattice Model'), and as the discrete time intervals becoming smaller and smaller, the discrete model should converge to the continuous model. As the option price at any date will be zero if 'out-of-the-money' ( $S < X$  for a CO) with a zero tv (no value is gained by delaying the exercise), these dates are ignored in the option valuation. So the probability of the option being in-the-money at the relevant date is a key component of the model, as is the probability of a greater payoff being available at a future date due to the time value.

### One Period

#### *Binomial Model (BM)*

##### *BM Methodology*

The BM and BSM calculate prices by discounting at the risk free rate. In the BM it is assumed that, over a single time step, the current price of an asset ( $S_0$ ) can increase by a factor 'u' ( $S_{1u} = u.S_0$ ) or decrease by a factor 'd' ( $S_{1d} = d.S_0$ ) with true probabilities of  $p^*$  and  $1 - p^*$ , respectively (the sum of all probabilities of possible states at any time must equal 1).  $S_0$  will therefore be the expected price at the next step  $S_1$  (probability weighted price) discounted at the risk-adjusted rate (i.e. cost of equity). In order to value the

option payoff using the risk-free rate, an adjustment must be made to the true probabilities to ensure the up and down states are 'Certainty Equivalents' / 'CE':

$$\frac{\text{Expected price} \times \text{true probability}}{1 + \text{risk adjusted rate}} = \frac{\text{CE price} \times \text{'Risk Neutral' probability}}{1 + \text{risk free rate}}$$

$$S_0 = \frac{u \cdot S_0 \cdot p^* + d \cdot S_0 (1-p^*)}{1 + R_{fd} + \text{risk premium}} = \frac{u \cdot S_0 \cdot p + d \cdot S_0 (1-p)}{1 + R_{fd}}$$

Where:

$p^*$  true probability

$p$  risk neutral probability (not a real probability but an adjusted true probability)

$R_{fd}$  risk free rate (discrete rate)

Taking a simple one step / two state example, assuming the exercise price is the grant date market price ( $X = S_0 = 150.00$ ), time to expiry 1 year, volatility 40.0%, the discrete risk free rate 5.13%<sup>1</sup> and the risk premium 2.98% (in CAPM this would equal the geared beta x market equity risk premium), using the approach of Cox, Ross & Rubinstein (1979) and ignoring dividends for now, the up and down factors  $u = 1.4918 (= e^{(40\% \times \sqrt{1.0})})$  and  $d = 0.6703 (= 1 / u)$ , would give up and down state asset prices and intrinsic values of  $S_{u1} = 223.77$  and  $S_{d1} = 100.55$ . Assuming these up and down states have an equal true probability ( $p^* = 0.5$ ), we can compute the risk neutral probability as  $p^* = 0.4637$ :

$$150 = \frac{(1.4918 \times 150 \times 0.5 + 0.6703 \times 150 \times (1 - 0.5))}{1 + 5.13\% + 2.98\%} = \frac{(1.4918 \times 150 \times 0.4637 + 0.6703 \times 150 (1 - 0.4637))}{1 + 5.13\%}$$

$$150 = \frac{162.16}{1 + 8.11\%} = \frac{150.68}{1 + 5.13\%}$$

where

$$\begin{aligned} p^* &= \frac{(1 + R_{fd} + u)}{(u - d)} \\ &= \frac{(1 + 5.13\% - 0.6703)}{(1.4918 - 0.6703)} \\ &= 0.4637 \end{aligned}$$

The expected price using risk neutral probabilities (150.68) is the certainty equivalent price.

The up and down factors depend on how volatile the asset price is expected to be over the period to expiry (measured as the standard deviation of returns). The more volatile the asset returns, the greater the likelihood the asset price will increase above the exercise price (for a CO), increasing the option FV. How this volatility is incorporated in u and d depends on the method chosen (the CRR method is used above, but there are others such as Jarrow and Rudd (1983)).

<sup>1</sup> The relationship between the discrete rate ( $R_{fd}$ ) and continuous rate ( $R_{fc}$ ) is  $R_{fd} = (e^{R_{fc}} - 1)$  and  $R_{fc} = \ln(1 + R_{fd})$  where 'e' is the exponential function (2.71828...) and 'ln' the natural logarithm. For example, a 5.0% nominal rate continuously compounded would give an effective annual rate of 5.1271%

Period	Year	Compounded	Continuou:	Effective
Annual	1	1	5.0000%	5.0000%
Semi-annual	0.5	2	5.0000%	5.0625%
Quarterly	0.25	4	5.0000%	5.0945%
Monthly	0.0833333	12	5.0000%	5.1162%
Weekly	0.0192308	52	5.0000%	5.1246%
Daily	0.0027397	365	5.0000%	5.1267%
Hourly	0.0001142	8,760	5.0000%	5.1271%
Every minute	0.0000019	525,600	5.0000%	5.1271%

$$(1 + 5.0000\% / 525,600)^{525,600} - 1 = 5.1271\%$$

$$\text{EXP}(5.0000\%) - 1 = 5.1271\%$$

### *BM Single Step Option Value (no dividends)*

The expected price using risk neutral probabilities would suggest a modest IV of 0.68 (=150.68 – 150.00), however this ignores the fact that the down state price would have zero IV as the call option would be worthless (out-of-the-money). The asset prices and IV at the up and down states using risk neutral probabilities are:  $S_{u1} = 223.77$  (=150.00 x 1.4918),  $IV_{u1} = 73.77$  (=223.77 – 150.00),  $S_{d1} = 100.55$ ,  $IV_{d1} = 0$  (= max {0, 100.55 – 150.00}). The down state price is ignored in the valuation, so that the option value is calculated as:

$$32.54 = \frac{IV_u P_u + IV_d P_d}{1 + R_{fd}} = \frac{73.77 \times 0.4637 + 0.00 \times (1 - 0.4637)}{1 + 5.13\%}$$

## Black Scholes Model (BM)

### BSM Methodology

The well know BSM (a derivation can be found elsewhere) is the continuous time version of the BM, and attempts, like the BM, to estimate future probability weighted expected prices and expected positive payoffs. As the number of time periods in the BM increases, the option value converges to the BSM (discussed below). The BSM uses logarithmic returns and defines the relationship between the current price and exercise price in terms of  $\ln(S_0 / X)$ , adjusting this to estimate the forward value by the risk free rate of return:

*Black-Scholes Model (no dividends)*

$$\text{Call option value} = S N(d_1) - X e^{-rT} N(d_2)$$
$$\text{Put option value} = X e^{-rT} N(-d_2) - S N(-d_1)$$

S      Current stock price  
X      Exercise Price  
T      Time to maturity  
r      Risk free rate (continuously compounded)  
e      The base of natural logarithms, constant = 2.71828  
 $X e^{-rT}$       The amount of cash needed to be invested over a period of time T at a continuously compounded interest rate r in order to receive X at maturity (i.e. it is the PV of the exercise price continuously discounted at the risk free rate).

$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(S/X) + rT}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T}$$
$$d_2 = \frac{\ln(S/X) + (r - 0.5\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where  $\sigma$  is the annualised standard deviation of logarithmic stock returns

$N(x)$       The cumulative probability function for a standardised normal variable.  $N(d_1)$  and  $N(d_2)$  will be discussed later, in the context of the Binomial model.

### BSM Single Step Option Value (no dividends)

Using the example for the BM, we can value the option assuming exercising occurs after 1 year. The call option value is 27.0344, lower than the BM because the BSM assumes continuous compounding over the 1 year whereas the BM is a discrete version with one compounding:

Current share price (S)	150.00	Time to Expiry (years)(T)	1.00 yrs
Exercise Price (X)	150.00	Risk Free Rate (%) (discrete)(r)	5.13%
Volatility (%)(standard deviation)	40.00%	Risk Free Rate (%) (continuous)(r)	5.00%
$d1 = (\ln(150.00 / 150.00) + (5.00\% - 0.00\% + 0.5 \times 40.00\%^2) \times 1.00) / (0.5 \times 40.00\% \times \text{SQRT}(1.00))$			0.325000
$d2 = 0.3250 - 40.00\% \times \text{SQRT}(1)$			-0.075000
$N(d1) = \text{NORMSDIST}(0.3250)$			0.6274
$N(d2) = \text{NORMSDIST}(-0.0750)$			0.4701
Share price ( 150.00 ) x N(d1)			94.11
PV of ex price { 150.00 x EXP(-5.00 x 1) } x N(d2)			67.08
Call Price			<b>27.0344</b>

To equate the BM to the BSM, the time steps in the BM need to be increased significantly, which requires the use of a multi-step tree.

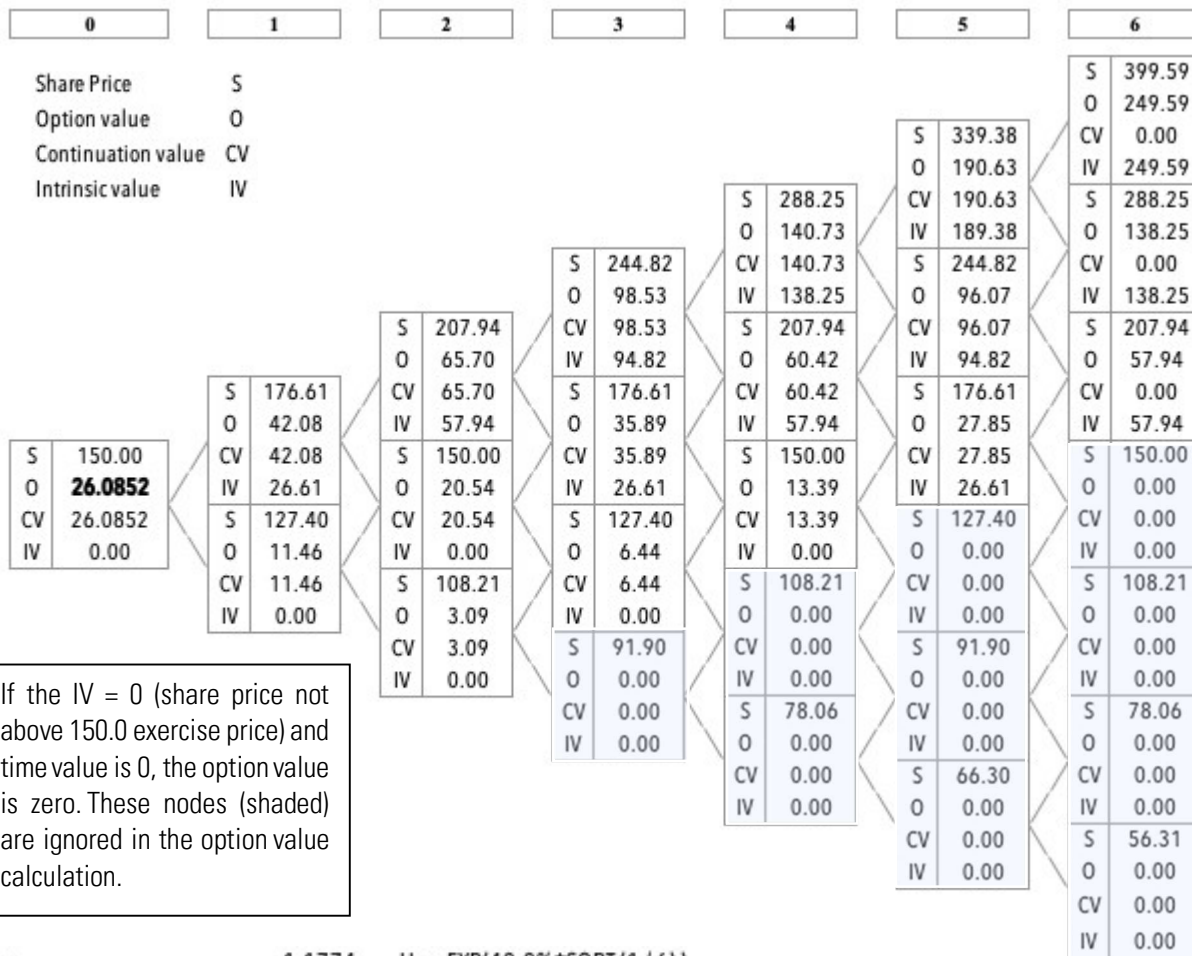
### Multi-period Binomial Trees

A 'Binomial Tree' (or 'Lattice') shows expected stock prices, Intrinsic Values and option values for each period until expiry. At the end of each stock price path (a 'node' – for example, the up and down state nodes in the above single period example), the stock price can increase or decrease at the next time period, so that nodes multiply geometrically (trinomial models add a third path between the up and down binomial states). If the option can be exercised at any date before expiry (American option), the value at each node will be the Intrinsic Value at that node or, if higher because of time value, the present value of the option at the next period (the 'Continuing Value'):

$$\text{Continuing Value}_t = \frac{p \times (\text{Option Value}_{t+1u}) + (1 - p) \times (\text{Option Value}_{t+1d})}{1 + \text{risk free rate } (R_{fd})}$$

At the final time period  $t_n$  (the expiry), the option value will simply be  $\max\{0, S_n - X\}$  i.e. the IV or zero as for the one period example above.

For a European option, the value at any time period before expiry will be the Continuing Value, since it cannot be exercised until expiry. Using the same information, but assuming the 1 year period is divided into 6 time steps, the following tree and an option value (26.0852) is shown:



u	1.1774	U = EXP(40.0%*SQRT(1 / 6))
d	0.8493	D = 1/1.1774
Risk-free return	1.0084	r = EXP(5.00%*0.1667)
Risk-neutral probability	0.4848	p = (1.0084 - 0.8493)/(1.1774 - 0.8493)

	<i>Example of workings - cell U<sup>4</sup></i>
S = S <sub>t-1</sub> x U	288.25 = 244.82 x 1.1774
O = MAX (CV, IV)	140.73 = MAX{140.73, 138.25}
CV = [ pC <sub>u</sub> + (1-p)C <sub>d</sub> ] / (1+r)	140.73 = [ 0.4848 x 190.63 + (1-0.4848) x 96.07 ] / 1.0084
IV = Stock Price - Exercise Price	138.25 = 288.25 - 150.00

A European Call Option on a non-dividend paying stock will have the same value as its American counterpart, since it would never be optimal to exercise the latter early (this is not the case if dividends are paid).

## BM-BSM Convergence

The call option value calculated above (26.0852) can be calculated using the Binomial distribution, a discrete probability distribution which measures the number of successes (probability 'p') and failures (probability '1-p') in a sequence of independent trials.

The call option can be valued as:

$$C_0 = S_0 \times \sum_{n=a}^T \left\{ A \times \left( \frac{u^n d^{T-n}}{(1+r)^T} \right) \right\} - \frac{X}{(1+r)^T} \times \sum_{n=a}^T \left\{ A \right\}$$

PV of expected stock price at each node, where exercising the option is optimal (probability weighted).

PV of exercise price at each node where exercising is optimal (probability weighted).

Where:

$$A = \frac{T!}{n!(T-n)!} p^n (1-p)^{T-n} = \text{possible combinations} \times \text{probability of success}$$

Each node can be reached via  $T! / n!(T-n)!$  possible paths.

Node 'n'	p	p <sup>n</sup>	(1-p) <sup>T-n</sup>	P <sub>u</sub> x P <sub>d</sub>	Paths	Binomial probability	In-the-money?	Prob of exercise	State Price	Yr 6 price (S <sub>T</sub> )	PV of Expected value of S <sub>T</sub> / S <sub>0</sub>
A	B	C = A X B	D	E = C X D	F	G = E X F		A	A / e <sup>r</sup>	u <sup>n</sup> x d <sup>(T-n)</sup> B	A x B / e <sup>r</sup>
6	0.4848	0.0130	1.0000	0.0130	1	0.0130	✓	0.0130	0.0123	2.6639	0.0329
5	0.4848	0.0268	0.5152	0.0138	6	0.0828	✓	0.0828	0.0787	1.9217	0.1513
4	0.4848	0.0552	0.2655	0.0147	15	0.2199	✓	0.2199	0.2092	1.3862	0.2900
3	0.4848	0.1139	0.1368	0.0156	20	0.3116	✗				
2	0.4848	0.2350	0.0705	0.0166	15	0.2484	✗				
1	0.4848	0.4848	0.0363	0.0176	6	0.1056	✗				
0	0.4848	1.0000	0.0187	0.0187	1	0.0187	✗				
						<b>1.0000</b>		<b>0.3157</b>			<b>0.4742</b>

Node 4 = (0.4848<sup>4</sup>) x (1 - 0.4848)<sup>(5 - 4)</sup> = 0.0147

Exercise Price		150.00	Share Price	150.00
Discount factor	x	0.9512		
Probability of exercise	x	0.3157	Payoff factor	0.4742
		45.0389		71.1240
			<b>Value 26.0852</b>	

0	1	2	3	4	5	6	Node 'n'	Share price S <sub>i</sub>	Return (S <sub>i</sub> /S <sub>0</sub> ) - 1	Return ln(S <sub>i</sub> /S <sub>0</sub> )	
S 150.00	U S 176.61 D S 127.40	UU S 207.94 UD S 150.00 DD S 108.21	UUU S 244.82 UUD S 176.61 UDD S 127.40 DDD S 91.90	UUUU S 288.25 UUUD S 207.94 UUDD S 150.00 UUDD S 127.40 UUDD S 108.21 UUDD S 78.06	UUUUU S 339.38 UUUUD S 244.82 UUUDD S 176.61 UUDDD S 127.40 UUDDD S 108.21 UUDDD S 91.90 UUDDD S 66.30	UUUUUU S 399.59 UUUUUD S 288.25 UUUUDD S 207.94 UUUUDD S 150.00 UUUUDD S 108.21 UUUUDD S 78.06 UUUUDD S 56.31	6	399.59	166.4%	98.0%	
							5	288.25	92.2%	65.3%	
							4	207.94	38.6%	32.7%	
							3	150.00	0.0%	0.0%	
							2	108.21	-27.9%	-32.7%	
							1	78.06	-48.0%	-65.3%	
							0	56.31	-62.5%	-98.0%	
								<i>Discrete</i>		<i>Continuous</i>	
								Mean μ	157.69	5.13%	-2.98%
								Variance σ <sup>2</sup>	4214.13	18.7%	16.0%
								σ	64.92	43.3%	40.0%
								r - 0.5v		-3.000%	

The Excel function can also be used:

Up factor	1.1774	
Down factor	0.8493	
Min up moves before in-the-money (integer)	3.0000	= INT{LN [ 150 [X]/ 150 [So] x 0.8493 ^6 ] / LN [ 1.1774 / 0.8493 ] }
Total trials	6	Time steps
Risk neutral probability	0.4848	
Risk free return per time step	1.0084	r = EXP(5%*0.1667)
Probability of success	0.5660	= 0.4848 x ( 1.1774 / 1.0084 )
Current share price factor	0.4742	= 1 - BINOMDIST(3, 6, 0.5660, TRUE)
Exercise price factor	0.3157	= 1 - BINOMDIST(3, 6, 0.4848, TRUE)
Call Price	<b>26.0852</b>	= 150 x 0.4742 - 150 / 1.0084^(6) x 0.3157

TRUE' is a signal to Excel that the distribution must be cumulative (i.e. probability higher than X)

The number of nodes can be increased to check convergence to the BSM, here up to 1000 time steps:

EUROPEAN CALL OPTION - BINOMIAL - 1, 2, 3, ..., 1000 TIME PERIODS - 1 YEAR (EXTRACTS)									
Time Steps in 1 year	Up	Down	1 + r	Risk neutral p	Min up moves	P* of success	P. P(x)	P	Binomial Call
1	1.491825	0.670320	1.0513	0.463724	0	0.658055	0.6581	0.4637	<b>32.5421</b>
2	1.326896	0.753638	1.0253	0.473917	1	0.613313	0.3762	0.2246	24.3763
3	1.259784	0.793787	1.0168	0.478586	1	0.592949	0.6378	0.4679	28.9109
4	1.221403	0.818731	1.0126	0.481403	2	0.580683	0.4421	0.2851	25.6323
5	1.195884	0.836202	1.0101	0.483339	2	0.572266	0.6336	0.4688	28.1554
6	1.177389	0.849337	1.0084	0.484774	3	0.566031	0.4742	0.3157	<b>26.0852</b>
7	1.163213	0.859688	1.0072	0.485893	3	0.561174	0.6318	0.4692	27.8324
8	1.151910	0.868123	1.0063	0.486796	4	0.557251	0.4939	0.3348	26.3174
9	1.142631	0.875173	1.0056	0.487546	4	0.553998	0.6308	0.4694	27.6536
996	1.012755	0.987405	1.0001	0.498812	497	0.505149	0.6394	0.4827	27.0286
997	1.012749	0.987412	1.0001	0.498812	498	0.505146	0.6274	0.4701	27.0400
998	1.012742	0.987418	1.0001	0.498813	499	0.505144	0.6154	0.4575	27.0286
999	1.012736	0.987424	1.0001	0.498814	499	0.505141	0.6274	0.4701	27.0399
1000	1.012729	0.987431	1.0001	0.498814	500	0.505139	0.6154	0.4575	<b>27.0286</b>

\* Workings - time step 999:

Up	= EXP(40.00% x SQRT(1 / 999))	= 1.012736	Down = 1 / U
1 + r*	= EXP(5.00% x SQRT(1 / 999))	= 1.0001	
p*	= (1.0001 - 0.9874) / (1.0127 - 0.9874)	= 0.498814	
Min moves	= INT(LN(150.0 / (150.0 x (0.9874 ^ 999))) / LN(1.0127 / 0.9874))	= 499	
Pr success	= 0.4988 x 1.0127 / 1.0001	= 0.505141	
(P)(Px)	= (1 - BINOMDIST(499, 999, 0.5051, TRUE))	= 0.6274	
P	= (1 - BINOMDIST(499, 999, 0.4988, TRUE))	= 0.4701	
Binom Call	= 0.6274 x 150.0 - 0.4701 x 142.68 (the PV of the exercise price)	= 27.04	

## Dividends

Dividends reduce the value of a call option since the option holder does not receive the cash payout, which is perceived to reduce equity value marginally. One method of incorporating dividends into the BM is by assuming they are paid at a continuous yield (q%) and adjusting downwards the risk neutral probability to  $P = (e^{(r-q)t} - d) / (u - d)$ . Using the 6 period example, assuming a 6.00% annual dividend yield (1.00% each time step), the option value reduces from 26.0852 to 22.1798 for a European option and 22.3702 for an American option. In the latter case, it is optimal to exercise early (the layout has been rotated 45 degrees for presentation purposes)

European							American								
	0	1	2	3	4	5	6		0	1	2	3	4	5	6
S	150.00	176.61	207.94	244.82	288.25	339.38	399.59	S	150.00	176.61	207.94	244.82	288.25	339.38	399.59
IV	0.00	0.00	0.00	0.00	0.00	0.00	249.59	IV	0.00	26.61	57.94	94.82	138.25	189.38	249.59
TV	22.18	37.36	60.67	94.07	137.75	189.13	0.00	TV	22.37	11.07	3.23	0.00	0.00	0.00	0.00
O	<b>22.1798</b>	<b>37.36</b>	<b>60.67</b>	<b>94.07</b>	<b>137.75</b>	<b>189.13</b>	<b>249.59</b>	O	<b>22.3702</b>	<b>37.68</b>	<b>61.17</b>	<b>94.82</b>	<b>138.25</b>	<b>189.38</b>	<b>249.59</b>
	S	127.40	150.00	176.61	207.94	244.82	288.25		S	127.40	150.00	176.61	207.94	244.82	288.25
	IV	0.00	0.00	0.00	0.00	0.00	138.25		IV	0.00	0.00	26.61	57.94	94.82	138.25
	TV	9.48	17.85	32.69	57.44	94.57	0.00		TV	9.57	18.01	6.37	0.00	0.00	0.00
	O	<b>9.48</b>	<b>17.85</b>	<b>32.69</b>	<b>57.44</b>	<b>94.57</b>	<b>138.25</b>		O	<b>9.57</b>	<b>18.01</b>	<b>32.98</b>	<b>57.94</b>	<b>94.82</b>	<b>138.25</b>
		S	108.21	127.40	150.00	176.61	207.94			S	108.21	127.40	150.00	176.61	207.94
		IV	0.00	0.00	0.00	0.00	57.94			IV	0.00	0.00	0.00	26.61	57.94
		TV	2.48	5.46	11.99	26.36	0.00			TV	2.51	5.51	12.11	0.00	0.00
		O	<b>2.48</b>	<b>5.46</b>	<b>11.99</b>	<b>26.36</b>	<b>57.94</b>			O	<b>2.51</b>	<b>5.51</b>	<b>12.11</b>	<b>26.61</b>	<b>57.94</b>
			S	91.90	108.21	127.40	150.00				S	91.90	108.21	127.40	150.00
			IV	0.00	0.00	0.00	0.00				IV	0.00	0.00	0.00	0.00
			TV	0.00	0.00	0.00	0.00				TV	0.00	0.00	0.00	0.00
			O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>				O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
	S	Share Price	S	78.06	91.90	108.21				S	78.06	91.90	108.21		
	IV	Intrinsic Value	IV	0.00	0.00	0.00				IV	0.00	0.00	0.00	0.00	
	TV	Time Value	TV	0.00	0.00	0.00				TV	0.00	0.00	0.00	0.00	
	O	Option Value	O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>				O	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	
			S	66.30	78.06						S	66.30	78.06		
			IV	0.00	0.00						IV	0.00	0.00		
			TV	0.00	0.00						TV	0.00	0.00		
			O	<b>0.00</b>	<b>0.00</b>						O	<b>0.00</b>	<b>0.00</b>		
			S	56.31							S	56.31			
			IV	0.00							IV	0.00			
			TV	0.00							TV	0.00			
			O	<b>0.00</b>							O	<b>0.00</b>			

A European option cannot be exercised before expiry, so IV will be zero until the final node

Shaded nodes are ignored, as there is payoff

Model assumptions	
Share price today	150.00
Exercise Price	150.00
Risk Free interest rate	5.00%
Volatility	40.0%
Expiry date	1.00 yrs
Time step (fraction of year)	1/6
Call (c) or Put (p)	c
Dividend yield per year	6.0%
Dividend yield per period	1.00%

Parameters	
u	1.1774 = EXP(40.0%*SQRT( 1 / 6))
d	0.8493 = 1/1.1774
r	0.9983 = EXP(5.00% - 1.00%) x (1 / 6)
p	0.4542 = (0.9983 - 0.8493)/(1.1774 - 0.8493)

The BSM incorporates a continuous dividend yield by reducing the risk free rate:

Current share price (S)	150.00	Time to Expiry (years)(T)	1.00 yrs
Exercise Price (X)	150.00	Risk Free Rate (%) (discrete)(r)	5.13%
Volatility %(standard deviation)	40.00%	Risk Free Rate (%) (continuous)(r)	5.00%
Continuous dividends - yield %(q)			<b>6.00%</b>
Share Price			150.00
Share price adjusted to $150.00 \times \text{EXP}(6.00\% \times 1)$			141.26
$d1 = (\text{LN}(150.00 / 150.00) + (5.00\% - \text{6.00}\% + 0.5 \times 40.00\%^2) \times 1.00) / (0.5 \times 40.00\% \times \text{SQRT}(1.00))$			0.175000
$d2 = 0.1750 - 40.00\% \times \text{SQRT}(1)$			-0.225000
$N(d1) = \text{NORMSDIST}(0.1750)$			0.5695
$N(d2) = \text{NORMSDIST}(-0.2250)$			0.4110
Share price ( 141.26 ) x N(d1)			80.44
PV of ex price $\{150.00 \times \text{EXP}(-5.00 \times 1)\} \times N(d2)$			58.64
Call Price			<b>21.8028</b>

## Appendix 2 : Convertible Bond Pricing

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Building the convertible binomial tree would typically involve the following steps:

1. Value the bond as a straight bond without any conversion feature (see Part 4).
2. For each coupon payment period until maturity, forecast share prices using techniques described in Appendix 1 (applying up and down factors to the prior node ex-dividend share price) and deduct any dividend assumed (this could be a yield %) to arrive at the ex-dividend share price (the yield can alternatively be deducted in the risk neutral probability calculation). This gives the Conversion Value (ex. div.share price x Conversion Ratio).
3. At the last node (maturity date), calculate the Investment Value (the redemption amount plus final coupon). At the previous node discount this back at the risk adjusted rate and add the coupon to determine the prior period Investment Value. Carry on backwards until time 0.
4. At the last node, the Convertible fair price will be the greater of the Conversion Value (when conversion is certain and the bond would be priced as 100% equity) and the Investment Value (when the bond is priced as 100% debt).
5. At the penultimate node, the convertible price will be the greater of the Conversion Value, the Investment Value and the present value of the Convertible fair price if conversion occurred at some future date (the 'Continuing Value', 'V' or 'Rollback' value). V is the present value of the risk-neutral probability weighted convertible fair price ('C') at the next period:

$$\text{Continuing Value } V_t = \frac{p \cdot C_{t+1u} + (1-p) \cdot C_{t+1d}}{1+r} + \text{coupon}$$

where  $t+1u, t+1d$  up and down states at next node after node  $t$

$p$  risk-neutral probability

$r$  discount rate

The same calculation is then carried out at the previous node and all others, going backwards.

There are two other factors to consider

- As discussed in the main text, issuers will usually require the right to call the bonds. If the convertible fair price or trading price (if quoted) rises above the call price (as yields fall), the bonds could be redeemed ('hard call') and refinanced at a lower coupon at less cost than buying them back in the market. In practice, issuers may call the bonds to force bond investors to convert if this is the best action to take (compared to doing nothing or having the bonds redeemed).

The convertible fair price should reflect the higher of the Conversion Value and the lower of the Continuing Value (if not converted or called) and Call Price (a call is only likely if the Call Price is less than the Continuing Value):

$$\text{Convertible fair price} = \text{MAX} ( \text{Conversion Value}, \text{MIN} \{ \text{Continuing Value}^*, \text{Call Price}^* \} )$$

\* plus Coupon

A notice to call the bonds should force the investor to convert and receive the higher cash payout.

- As the Convertible is a hybrid instrument whose value reflects the underlying investment value (which acts as a floor) and equity value (with upside potential), its fair price could be calculated with suitable discount rates that reflect these two components. As in the option pricing model, the equity component can be discounted at the risk free rate; the discount rate for the debt portion should reflect the issuer’s credit risk and would therefore be the risk free rate plus a credit risk premium. Two approaches that have been suggested in the past are as follows:

- Discount the next period probability weighted equity and debt components at the risk free and risk adjusted rates, respectively (this requires working backwards from the final node, when the bond will be priced 100% debt or equity, and discounting back using these rates to arrive at the equity and debt values at the prior node etc):

$$V_t = \frac{p \cdot E_{t+1u} + (1-p) \cdot E_{t+1d}}{1 + r} + \frac{p \cdot D_{t+1u} + (1-p) \cdot D_{t+1d}}{1 + r + \text{CRP}}$$

Where:

- t+1u, t+1d up and down states at next node after node t
- p risk-neutral probability
- r risk free rate
- CRP credit risk premium

(Tsiveriotis and Fernandes (1998) and Hull (2003))

- Discount the next period probability weighted convertible fair value (equity and debt) at a blended rate that varies as the equity and debt components change, respectively:

$$\text{Blended rate} = r \cdot w + (r + \text{CRP})(1 - w)$$

Where:

- r risk free rate
- CRP Credit Risk Premium

$w$  a measure of the equity component embedded in the next period convertible fair price, which can be estimated using the convertible's 'delta' or the probability of conversion:

$$[1] \text{ Delta} = \frac{\text{Convertible Price}_u - \text{Convertible Price}_d}{\text{Conversion Value}_u - \text{Conversion Value}_d}$$

(Woodson (2002), Philips (1997))

$$[2] \text{ Prob. of conversion} = P_u \cdot p + P_d \cdot (1 - p)$$

$P_u, P_d$  probability of conversion at next period up and down states

$p$  risk neutral probability

(Goldman Sachs (1994))

This simplified example illustrates pricing a convertible using the blended rate approach based on the probability of conversion discussed above. Assume the following:

A 2.00% p.a. coupon paying bond, maturing in 6 years at 129.318% of face value, is issued at €115 with an option to convert into 5 shares at any year end at a €20.00 Conversion Price (39.74% premium to the €14.31 share price on issue). The issuer has the right to redeem the bond early (call) at amortised cost from year 4. The yield for a similar bond without any conversion feature is 6.184% (effective rate), giving a PV of €100 for the straight bond less 4.15 for call option = 95.85 + 19.15 for the conversion option (volatility is 40.0%).

Issue Date			Thu 31 Jul 2025
Valuation Date			Thu 31 Jul 2025
Maturity date			Thu 31 Jul 2031
Maturity			6.00 years
Principal Amount (Face Value)	FV	€	100.000
Redemption Amount	RA	€	129.318
Coupon			2.000%
Coupon dates (1 = Annual, 2 = Semi-Annual)			1
Next coupon - first period			Fri 31 Jul 2026
Accrued interest		€	0.0000
Yield per coupon period		Effective	6.184%
Number of shares in issue			500,000,000
Number of bonds in issue			200,000
Share Price	S	€	14.31
Equity Volatility			40.0%
Conversion Premium	P		39.74%
Conversion Price = $S \times (1 + P)$	CP		20.0000
Conversion Ratio (Parity) = $FV / CP$	CR		5.00 shares
Risk Free Rate (nominal)	$R_f$	Continuous	5.000%
Credit Risk Premium	CRP	Continuous	1.000%
Time steps	$t$		6.00
Time step (fraction of year)			1.00
Dividend yield			1.00%
Discount Conv Price at blended rate <sup>1</sup> or Debt/Equity <sup>2</sup>			Blended Rate
Blended rate using Delta or Probability of Conversion			Probability
Coupon added to Convertible Price?			Yes
Dividend yield per period			1.00%

<sup>1</sup> Goldman Sachs (1994), <sup>2</sup> Tsiveriotis & Fernandes (1998)

Date	Period	Convertible	Callable	Call Price	Dividend Yield
31-Jul-26	1	Yes		-	1.00%
31-Jul-27	2	Yes		-	1.00%
31-Jul-28	3	Yes		-	1.00%
31-Jul-29	4	Yes	Yes	118.352	1.00%
31-Jul-30	5	Yes	Yes	123.670	1.00%
31-Jul-31	6	Yes	Yes	129.318	1.00%

The first step is to calculate the straight bond value (Investment Value), assuming no conversion. As the valuation date is the bond issue date, accrued interest is zero:

Time	Date	Cash Flows	Disc F 6.184%	PV at $t_0$	PV at at $t_n$	Call Price (excl.coupon)
0	31-Jul-25				100.0000	
1	31-Jul-26	2.0000	0.9418	1.884	106.1837	104.1837
2	31-Jul-27	2.0000	0.8869	1.774	110.6260	108.6260
3	31-Jul-28	2.0000	0.8353	1.671	115.3431	113.3431
4	31-Jul-29	2.0000	0.7866	1.573	120.3518	118.3518
5	31-Jul-30	2.0000	0.7408	1.482	125.6703	123.6703
6	31-Jul-31	131.3176	0.6977	91.617	131.3176	129.3176
				<u>100.000</u>		

Now the tree:

#### BINOMIAL PARAMETERS

Risk Free per step	= $\text{EXP}(5.00\% \times 1.0) - 1$	5.13%
Credit Risk Adjusted per step	= $\text{EXP}(6.00\% \times 1.0) - 1$	6.18%
1 + Risk-Free Rate	= 1 + 5.1271%	1.0513
1 + Risk-Adjusted Rate	= 1 + 6.1837%	1.0618
Up-move factor (u)	= $\text{EXP}(40.0\% \times \text{SQRT } 1.00)$	1.4918
Down-move factor (d)	= 1/1.4918	0.6703
Risk-Neutral Probability	= $(1.0513 - 0.6703)/(1.4918 - 0.6703)$	0.4637

The share price at each node of the tree is the ex-div price at the previous node adjusted up or down by the u and d factors. The dividend is calculated as a % yield of the price (which is assumed to be the cum-div price, from which the dividend is deducted to get the ex-div price). The alternative would be to adjust the risk neutral probability

	0	1	2	3	4	5	6
Date	31-Jul-25	31-Jul-26	31-Jul-27	31-Jul-28	31-Jul-29	31-Jul-30	31-Jul-31
Callable		No	No	No	Yes	Yes	Yes
1 + Risk Free	105.13%	105.13%	105.13%	105.13%	105.13%	105.13%	105.13%
1 + Risky	106.18%	106.18%	106.18%	106.18%	106.18%	106.18%	106.18%
Call Price					118.35	123.67	129.32
Div Yield		1.00 %	1.00 %	1.00 %	1.00 %	1.00 %	1.00 %
Straight bond value	100.000	106.184	110.626	115.343	120.352	125.670	131.318
Accrued interest / coupon		2.00	2.00	2.00	2.00	2.00	2.00
Conversion Ratio		5.00 shares	5.00 shares	5.00 shares	5.00 shares	5.00 shares	

	0	1A	1B	1C	1D	1E	
Share Price (Cum Div)	14.31	21.35	31.53	46.57	68.79	101.59	150.04
Dividend		0.21	0.32	0.47	0.69	1.02	1.50
Share Price (ex div)	14.31	21.14	31.22	46.11	68.10	100.57	148.54
Conversion Value	71.56	105.69	156.10	230.54	340.49	502.87	742.69
Debt Component	100.00	30.63	14.40				
Equity Component	15.00	105.69	156.10	230.54	340.49	502.87	742.69
<b>CONVERTIBLE PRICE</b>	<b>115.00</b>	<b>136.33</b>	<b>170.50</b>	<b>232.54</b>	<b>340.49</b>	<b>502.87</b>	<b>742.69</b>
Continuing value + Coupon	115.00	136.33	170.50	230.24	339.08	499.84	
Call Price + Coupon					120.35	125.67	131.32
Issuer action							
Holder action		Continue	Continue	Continue	Convert	Convert	Convert
Conversion Forced?							
Probability of Conversion	0.2601	0.4457	0.7124	1.0000	1.0000	1.0000	1.0000
Blended rate per period	5.91%	5.71%	5.43%	5.13%	5.13%	5.13%	5.13%

	2A	2B	2C	2D	2E	
Share Price (Cum Div)	9.59	14.17	20.93	30.91	45.65	67.42
Dividend	0.10	0.14	0.21	0.31	0.46	0.67
Share Price (ex div)	9.50	14.03	20.72	30.60	45.19	66.74
Conversion Value	47.49	70.14	103.59	152.99	225.95	333.71
Debt Component	61.74	47.22	26.60			
Equity Component	47.49	70.14	103.59	152.99	225.95	333.71
<b>CONVERTIBLE PRICE</b>	<b>109.23</b>	<b>117.36</b>	<b>130.19</b>	<b>152.99</b>	<b>225.95</b>	<b>333.71</b>
Continuing value + Coupon	109.23	117.36	130.19	165.78	225.69	
Call Price + Coupon				120.35	125.67	131.32
Issuer action						
Holder action		Continue	Continue	Convert	Convert	Convert
Conversion Forced?				Forced		
Probability of Conversion	0.0997	0.2150	0.4637	1.0000	1.0000	1.0000
Blended rate per period	6.08%	5.96%	5.69%	5.13%	5.13%	5.13%

	3B	3C	3D	3E	
Share Price (Cum Div)	6.37	9.40	13.89	20.51	30.29
Dividend	0.06	0.09	0.14	0.21	0.30
Share Price (ex div)	6.30	9.31	13.75	20.31	29.99
Conversion Value	31.52	46.55	68.74	101.53	149.95
Debt Component	110.63	115.34	120.35	125.67	
Equity Component					149.95
<b>CONVERTIBLE PRICE</b>	<b>110.63</b>	<b>115.34</b>	<b>120.35</b>	<b>125.67</b>	<b>149.95</b>
Continuing value + Coupon	110.63	115.34	120.35	133.81	
Call Price + Coupon			120.35	125.67	131.32
Issuer action					
Holder action		Continue	Continue	Continue	Convert
Conversion Forced?					
Probability of Conversion	0.0000	0.0000	0.0000	0.0000	1.0000
Blended rate per period	6.18%	6.18%	6.18%	6.18%	5.13%

	4C	4D	4E	
Share Price (Cum Div)	4.23	6.24	9.22	13.61
Dividend	0.04	0.06	0.09	0.14
Share Price (ex div)	4.18	6.18	9.12	13.48
Conversion Value	20.91	30.89	45.62	67.38
Debt Component	115.34	120.35	125.67	131.32
Equity Component				
<b>CONVERTIBLE PRICE</b>	<b>115.34</b>	<b>120.35</b>	<b>125.67</b>	<b>131.32</b>
Continuing value + Coupon	115.34	120.35	125.67	
Call Price + Coupon		120.35	125.67	131.32
Issuer action				
Holder action		Continue	Continue	Redeem
Conversion Forced?				
Probability of Conversion	0.0000	0.0000	0.0000	0.0000
Blended rate per period	6.18%	6.18%	6.18%	6.18%

The call option and conversion option have been netted off (15.0). Ignoring the call, the option is 19.15.

At maturity (t6), convertible holders will convert at level 3 and above, as the Conversion Value (ConvV) is greater than the Redemption Amount (RA). At year 5 (E nodes), the issuer can call (paying 125.67) but would only do so if this was less than the convertible fair value (without a call), being the maximum of the ConvV, Investment Value (IV) and Continuing Value (ContV) (with time value). At 1E and 2E, the investor would convert anyway (ConvV > ContV > RA). At 3E, the investor receives more from the call if made (ContV > RA > ConvV) but at 2D they receive more from converting compared to the call (ContV > ConvV > RA), so a call would force them to convert.

Continued on next page

	0	1	2	3	4	5	6
Date	31-Jul-25	31-Jul-26	31-Jul-27	31-Jul-28	31-Jul-29	31-Jul-30	31-Jul-31
Callable		No	No	Yes	Yes	Yes	Yes
1 + Risk Free	105.13%	105.13%	105.13%	105.13%	105.13%	105.13%	105.13%
1 + Risky	106.18%	106.18%	106.18%	106.18%	106.18%	106.18%	106.18%
Call Price		-	-	113.34	118.35	123.67	129.32
Div Yield		1.00 %	1.00 %	1.00 %	1.00 %	1.00 %	1.00 %
Straight bond value	100.000	106.184	110.626	115.343	120.352	125.670	131.318
Accrued interest / coupon	£ -	£ 2.00	£ 2.00	£ 2.00	£ 2.00	£ 2.00	£ 2.00
Conversion Ratio		5.00 shares	5.00 shares	5.00 shares	5.00 shares	5.00 shares	

Conversion option		19.15
Call option		(4.15)
Straight debt		100.00
Value of Convertible - undiluted	€	115.00
Conversion option	Equity	3,830,132
Debt (incl. call option)	Liability	19,169,879
Fair value (200,000 x 115.00)	€	23,000,011
Conversion option x1 / (1 + dilution factor)		18.19
Call option		(4.15)
Straight debt		100.00
Value of Convertible - diluted		114.04
New shares issued = Bonds x Conversion Ratio		26,263,525
Enlarged shares after conversion		526,263,525
Bond dilution factor (= Extra shares / Old shares)		5.25%

Share Price (Cum Div)	2.80	5D	4.14	5E	6.12
Dividend	0.03		0.04		0.06
Share Price (ex div)	2.78		4.10		6.05
Conversion Value	13.88		20.50		30.27
Debt Component	120.35		125.67		131.32
Equity Component	-		-		-
<b>CONVERTIBLE PRICE</b>	<b>120.35</b>		<b>125.67</b>		<b>131.32</b>
Continuing value + Coupon	120.35		125.67		-
Call Price + Coupon	120.35		125.67		131.32
Put Price	-		-		-
Issuer action	-		-		-
Holder action	Continue		Continue		Redeem
Conversion Forced?	-		-		-
Probability of Conversion	0.0000		0.0000		0.0000
Debt rate (risky)	6.18%		6.18%		6.18%
Equity rate (risk free)	5.13%		5.13%		5.13%
Blended rate	6.18%		6.18%		6.18%
Blended rate per period	6.18%		6.18%		6.18%

Share Price (Cum Div)	1.86	5E	2.75
Dividend	0.02		0.03
Share Price (ex div)	1.84		2.72
Conversion Value	9.21		13.60
Debt Component	125.67		131.32
Equity Component	-		-
<b>CONVERTIBLE PRICE</b>	<b>125.67</b>		<b>131.32</b>
Continuing value + Coupon	125.67		-
Call Price + Coupon	125.67		131.32
Put Price	-		-
Issuer action	-		-
Holder action	Continue		Redeem
Conversion Forced?	-		-
Probability of Conversion	0.0000		0.0000
Debt rate (risky)	6.18%		6.18%
Equity rate (risk free)	5.13%		5.13%
Blended rate	6.18%		6.18%
Blended rate per period	6.18%		6.18%

Share Price (Cum Div)	1.23
Dividend	0.01
Share Price (ex div)	1.22
Conversion Value	6.11
Debt Component	131.32
Equity Component	-
<b>CONVERTIBLE PRICE</b>	<b>131.32</b>
Continuing value + Coupon	-
Call Price + Coupon	131.32
Put Price	-
Issuer action	-
Holder action	Redeem
Conversion Forced?	-
Probability of Conversion	0.0000
Debt rate (risky)	6.18%
Equity rate (risk free)	5.13%
Blended rate	6.18%
Blended rate per period	6.18%

### Workings for node 1C

Share Price (Cum Div)	46.57	1C	= 31.22 x 1.4918	Prior Period Share Price (ex div) x Up factor
Dividend	0.47		= 46.57 x 1.00%	Share price at node x dividend yield
Share Price (ex div)	46.11		= 46.57 - 0.47	Share price - dividend
Conversion Value	V1	230.54	= 46.11 x 5.00	Share price (ex div) x Conversion Ratio = Value received on conversion
Debt Component		2.00		
Equity Component		230.54		100% equity as conversion will occur
<b>CONVERTIBLE PRICE</b>		232.54		Forced conversion, so value equals conversion value
Continuing value + Coupon	V2	230.24		= [0.4637 (RNP) x 340.49 (CP at 1D) + (1-0.4637) x 152.99 (CP at 2D)] / (1 + 5.13%) + 2.00
Holder action		Continue		Call price < Conversion value: Convert to avoid receiving less value
Probability of Conversion		1.0000		Conversion, so probability of conversion = 1
Debt rate (risky)		6.18%		Credit adjusted rate for period (applied to debt component)
Equity rate (risk free)		5.13%		Risk free rate for period (applied to equity component)
Blended rate per period		5.13%		As conversion will occur, the blended rate reflects equity risk only (hence risk free)

In general, the price is the higher of (1) Conversion Value (V1) and (2) lower of the Convertible Price ignoring any Call + coupon ('Rollback' value)(V2) and the Call Price +

