

SECURITY VALUATION

Option Pricing

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CONTENTS

Option Features	1
Pricing Approach.....	1
Pricing Relevance	2
Employee Stock Options	3
Features	3
Fair Value Estimation	3
Recognising the Option Expense under IFRS.....	4
Taxation of Options (UK)	5
Appendix 1: Option Pricing Models	7
Pricing without Dividends	7
Single Step	7
Binomial Model	7
Black-Scholes Model	9
Multi-Step.....	11
BM-BSM Convergence.....	12
Pricing with Dividends.....	15
Monte Carlo Simulation	17

Introduction

Financial risk management, employee remuneration, strategic investment decision making and the pricing of certain funding instruments may involve the use of options (interest rate options, employee stock options, real options and bonds with embedded options, respectively). This paper discusses basic option features ('plain vanilla' rather than 'exotic' options) and pricing methods ('Binomial Option Pricing', 'Black-Scholes Model', 'Simulation').

Option Features

In return for the payment of a 'premium' (the 'fair value' of the option), a purchaser becomes owner of an option that gives them the right (without any obligation) to buy ('Call' Option / 'CO') or sell ('Put' Option / 'PO') an asset, or lock into a rate (interest rate or currency exchange rate, for example) at a stated price or rate ('exercise' or 'strike' price) at some future specified date ('exercise date') before such a right expires ('expiry date'). The exercise date may be at any time up to and including expiry ('American' option), only on specified dates ('Bermudan' option) or on expiry ('European' option).

With a stock option, a CO holder would benefit if the stock price increased above the exercise price, sufficiently to cover the premium cost: the option would be exercised, the stock purchased at the lower exercise price and immediately sold at the higher market price (profit = sale price – exercise price – option premium, ignoring taxes). A fall in the stock price could make a profit for a PO holder: the stock would be purchased at the lower market price and immediately sold at the higher exercise price (profit = exercise price – stock price – option premium). The discussion that follows will concentrate on COs.

Pricing Approach

If a CO could only be exercised immediately after it was purchased, the premium ('P') paid to acquire the option would be its 'Intrinsic Value' ('IV'), $S - X$ (where S is the stock price and X the exercise price), so that the total cost to acquire the share ($P + X$) was the same as if purchased in the market (S). The option would be 'In-The-Money', since an immediate profit could be made (ignoring the premium). If the share price was less than the exercise price, the CO would be 'Out-Of-The-Money' and worthless.

¹ This is a new version of a paper originally dated 19 May 2026.

Of course, a CO would be purchased in the hope that an unlimited 'payoff' could be achieved as the stock price increased. At any given date during the term, the fair value of the option will equal its value if exercised immediately (IV) plus an extra value component ('Time Value') equal to the 'Present Value' / 'PV' of the expected incremental value from waiting until a later date in the hope the stock price rises:

$$\text{Option Premium (Value)} = \text{Intrinsic Value} + \text{Time Value}$$

The future payoff from delaying an exercise depends on how the stock price is expected to change over the term. If the stock price falls below the exercise price, its IV will be zero. An option pricing model forecasts future stock prices at various dates until the expiry date, assuming some probability distribution for those prices, and sets the payoff to zero in scenarios where the stock price is below the exercise price (when the option would not be exercised, and the loss suffered would be the premium paid).

Option pricing models, generally:

- forecast a range of stock prices at future exercise dates, assuming some probability distribution and degree of price variability (volatility);
- only consider future scenarios where the option is in the money (forecast prices that give a positive IV);
- calculate the PV of the future positive IV scenarios, weighted by the probability of their occurrence.

The two basic models take different approaches to forecasting stock prices, whether 'jumping' over some 'discrete' time interval (day, week, month etc) or continuously changing.

Pricing Relevance

The fair value of an option, as estimated by an option pricing model, is required for several purposes:

- Measurement of the grant-date fair value of employee options and other share-based payment transactions recognised under IFRS 2 for financial reporting purposes.
- Valuing a pool of existing options at the date a business is valued using DCF techniques, in order to allocate equity value to ordinary shareholders and option-holders.
- Valuing complex capital structures that have instruments with embedded options and different rights (see article 'Security Valuation – Using the Option Pricing Method for Complex Capital Structures').
- Determining the value attributable to options embedded in financial instruments, such as convertible bonds, convertible preference shares and callable bonds (see the article 'Security Valuation – Convertible Bond Pricing').

- Determining the value of any managerial decision-making flexibility when making capital investment decisions that involve expansion, delay or abandonment 'real' options.

Employee Stock Options

Features

Call Options may be granted to employees as part of their remuneration, with or without 'vesting' conditions that need to be fulfilled before the option 'vests'. Vesting conditions may require the employee to remain employed for a minimum length of service ('Service Conditions'). They may also have a condition that certain internal financial metrics are achieved ('Non-Market Performance Condition') or that the company share price reaches a stated level ('Market Performance Condition'). If these conditions are not met, the options are forfeited. If the options vest but are not exercised before the expiry (say 7 – 10 years), they lapse and are cancelled. Vesting can take place over time or immediately on some date ('cliff' structure).

The exercise price may be set at the fair value or market price of the underlying shares at the date they are granted ('at-the-money'), although it could be variable and depend on a measure of the stock price in the future (e.g. moving average or maximum over some period) or even zero. Exercising the option is usually the only way an employee can benefit financially from holding the option, since employee options are usually non-transferable and cannot be sold.

Fair Value Estimation

Estimating the fair value ('FV') of employee stock options using an option pricing model like the Binomial Model ('BM') (or 'Lattice' model) or Black-Scholes Model ('BSM') is made complicated by the following:

- The earliest date an employee can exercise the option may depend on contractual vesting conditions.
- The post-vesting date when the employee actually exercises the option will depend on when they decide to exercise, so an option model needs to estimate the expected term of the option based on an assumption about exercise behaviour. Options vested at the valuation date could be exercised immediately (if in-the-money), evenly between the valuation and expiry dates (on average at the mid-point), on expiry or according to some other method.
- Options may be grouped into 'cohorts' that share similar features, allowing valuation for the whole cohort without any material difference compared to pricing each option separately.

Option valuation may require an allowance for dilution, if exercising them creates additional shares. Damodaran's warrant model (<https://pages.stern.nyu.edu/~adamodar/pc/warrant.xls>) adjusts the model input share price to:

$$\text{Adjusted share price} = \frac{\text{shares outstanding} \times \text{share price} + \text{options} \times \text{value per option}}{\text{shares outstanding} + \text{options}}$$

The calculation is iterative because the adjusted stock price depends on the option value, which itself depends on that input. For example, assume a company with an equity DCF value per share of £10.00 (equity DCF £100m, outstanding shares 10m) grants 1m options with an average exercise price of £8.00 (5 years to expiry, 40% annual volatility, 5.0% risk-free rate, 1.00% continuous dividend yield), the option value calculated is £4.2268. The adjusted stock price to be input into the model is £9.4752.

Assumptions		
Current share price (S)	£	10.00
Shares outstanding	#	10,000,000
Equity Value	£	100,000,000
Exercise Price (X)	£	8.00
Options	#	1,000,000
Risk-free rate (R _{fc})	Continuous	5.00 %
Volatility	Standard Deviation	40.00 %
Time to Expiry (T)		5.00 yrs
Dividend yield (q)		1.00 %
Option Valuation		
Warrant adjusted share price (10m x £10.00 + 1m x £4.2268) / (10m + 1m) - circularity	£	9.4752
Share price adjusted to 9.4752 x EXP (- 1.00% x 5)	£	9.0131
d1 = (LN(9.48 / 8.00) + (5.00% - 1.00% + 0.5 x 40.00%^2) x 5.00) / (40.00% x SQRT(5.00))		0.8600
d2 = 0.8600 - 40.00% x SQRT(5)		-0.0344
N(d1)= NORMSDIST (0.8600)		0.8051
N(d2)= NORMSDIST (-0.0344)		0.4863
Share price (£9.01) x N(d1)	£	7.2565
PV of ex price {£8.00x EXP(-5.00% x 5 years) } x N(d2)	£	3.0297
Call Price	£	4.2268

Recognising the Option Expense under IFRS

Under IFRS 2, the FV of the equity-settled share-based payment is measured at the grant-date (use of the BM or BSM is permitted) and is not subsequently re-valued (cash-settled liabilities are remeasured). The option FV is expensed to profit and loss over the vesting period (with a corresponding increase in equity), depending on the proportion of options expected to vest (the allocation is intended to match the period over which the employee services are provided). If there are no vesting conditions, the option FV is immediately recognised in full.

Only Market Performance Conditions and non-vesting conditions are considered when estimating the option FV, typically using the BM or Monte Carlo simulation, as the BSM cannot normally accommodate them. Service and Non-Market Performance Vesting Conditions are incorporated via an adjustment to the number of options and do not affect grant-date FV ('modified grant-date' method) (IFRS 2 paras. 19-21A).

An estimate of the proportion of options expected to vest (for example, some employees may leave, so the options never vest) is made on each reporting date during the vesting period. In the first year the charge will be the vesting fair value (option FV at grant-date x options granted x % expected to vest) divided by the vesting period in years. If the expected vesting proportion is constant, the annual charge will be constant. If the second-year expectation changes, the second-year charge will differ (the first year charge is not affected), so a cumulative catch-up adjustment is made in that year. In the final year, an adjustment is made to ensure the actual number of options vesting is accounted for (see examples in EY (2026) §6.2.1.A ch.30).

Taxation of Options (UK)

In the UK, the granting of an option over shares would, if the qualifying conditions are met, enable the company to deduct an amount for corporation tax, but only if the option is exercised and the employee obtains beneficial ownership of the shares. Relief is given on 'an amount equal to the market value of the shares when they are acquired, less the total amount or value of any consideration given by any person in relation to the obtaining of the option or to the acquisition of the shares....' [s1018 Corporation Tax Act 2009] (although some 'restricted' and convertible securities have a different relief provision). 'The relief is given for the accounting period in which the shares are acquired...as a deduction in calculating the profits of the qualifying business for corporation tax purposes....' [s1021 Corporation Tax Act 2009] 'The statutory deduction overrides the accounting treatment' (HMRC Business Income Manual BIM44265). No deduction is available for the IFRS 2 accounting expense itself [s1038 Corporation Tax Act 2009].

The expense charged to the income statement for the fair value of employee stock option grants (allocated over the vesting period under IFRS 2) differs from the amount deductible for tax purposes, which will arise in later periods when the options are exercised (the tax deduction being the intrinsic value at the exercise date: share price less exercise price, as discussed above). There may be a mismatch between the accounting and taxable profits, requiring a deferred tax adjustment (see the article 'Business Acquisitions: Taxation (UK) & Reporting (IFRS)' for a discussion of deferred tax). A deductible temporary difference will arise as the carrying amount is nil and the tax base is the amount deductible in future periods for employee services rendered to the reporting date (the deferred tax asset equals the tax rate x tax base in this case) [see EY (2026) §10.8 ch.29].

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Suggested reading

Books:

Chriss, N.A. (1997) *Black-Scholes and Beyond: Option Pricing Models*. McGraw-Hill.
Clewlow, L. & Strickland, C. (1998) *Implementing Derivatives Models*, Wiley
Cox, J. C., and M. Rubinstein (1985) *Option Markets*. New Jersey: Prentice Hall.
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Rendleman Jr., R. J. (2002) *Applied Derivatives: Options, Futures, and Swaps*. Oxford: Blackwell Publishers
Rubinstein, M. (1999) *Rubinstein on Derivatives: Futures, Options and Dynamic Strategies*. London: Risk Books.
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Tan, P., Lim, C.Y., & Kuah, E.W. (2020) *Advanced Financial Accounting: An IFRS® Standards Approach* (4th ed.) McGraw-Hill

Papers

Black, F., and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81(3)
Cox, J., S. Ross, and M. Rubinstein (1979), "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7(3)
Merton, R.C. (1973), "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*, 4(1)

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The author wrote the article and built the Excel models. A.I. (Claude Opus 5 and Codex GPT-6 Astra) assisted with checking formulae, example arithmetic, cross references, citations and consistencies.

Pricing without Dividends

Single Step

Binomial Model

The most common discrete version of the models is the 'Binomial' Model ('BM'). In the BM it is assumed that, over a single time step, the current price of an asset (S_0) can increase by a factor 'u' ($S_{u1} = u.S_0$) or decrease by a factor 'd' ($S_{d1} = d.S_0$) with true probabilities of p^* and $1 - p^*$, respectively (the sum of all probabilities of possible states at any time must equal 1). S_0 will therefore be the expected price at the next step S_1 (probability weighted price) discounted at the risk-adjusted rate (i.e. cost of equity). In order to value the option payoff using the risk-free rate (used in the Black-Scholes Model), an adjustment is made to the true probabilities to obtain a market-value 'Certainty Equivalent' / 'CE':

$$\frac{\text{Payoff} \times \text{true probability}}{1 + \text{risk-adjusted rate}} = \frac{\text{Payoff} \times \text{'Risk-neutral' probability}}{1 + \text{risk-free rate}} \quad \text{Certainty Equivalent Price}$$

$$S_0 = \frac{u.S_0.p^* + d.S_0(1-p^*)}{1 + R_{fd} + \text{implied risk premium}} = \frac{u.S_0.p + d.S_0(1-p)}{1 + R_{fd}}$$

Where:

p^* true probability
 p risk-neutral probability
 R_{fd} Risk-free rate (discrete rate)

Taking a simple one step / two state example, assuming the exercise price is the grant-date market price ($X = S_0 = 150.00$), time to expiry 1 year, volatility 40.0%, and the discrete Risk-free rate 5.13%² (and an implied risk premium of 2.98% - equal to 0.662 geared beta x 4.50% market equity risk premium under CAPM – giving a required return on the share of 8.11%), using the approach of Cox, Ross & Rubinstein (1979) and ignoring dividends for now, the up and down factors $u = 1.4918$ ($= e^{40\% \times \sqrt{\Delta t}}$) and $d = 0.6703$ ($= 1 / u$), would give up and down asset prices of $S_{u1} = 223.77$ and $S_{d1} = 100.55$. Assuming these up and down states have an equal true probability ($p^* = 0.5$):

² The relationship between the discrete rate (R_{fd}) and continuous rate (R_{fc}) is $R_{fd} = (e^{R_{fc}} - 1)$ and $R_{fc} = \ln(1 + R_{fd})$ where 'e' is 2.71828 and 'ln' the natural logarithm. For example, a 5.0% nominal rate continuously compounded would give an effective annual rate of 5.1271%

Period	Year	Compounded	Continuou	Effective
Annual	1	1	5.0000%	5.0000%
Semi-annual	0.5	2	5.0000%	5.0625%
Quarterly	0.25	4	5.0000%	5.0945%
Monthly	0.0833333	12	5.0000%	5.1162%
Weekly	0.0192308	52	5.0000%	5.1246%
Daily	0.0027397	365	5.0000%	5.1267%
Hourly	0.0001142	8,760	5.0000%	5.1271%
Every minute	0.0000019	525,600	5.0000%	5.1271%
$(1 + 5.0000\% / 525,600)^{525,600} - 1 =$				5.1271%
$\text{EXP}(5.0000\%) - 1 =$				5.1271%

$$150 = \frac{162.16}{1 + 8.11\%} = \frac{157.69}{1 + 5.13\%}$$

$$p = \frac{(1 + R_{fd} - d)}{(u - d)} = \frac{(1 + 5.13\% - 0.6703)}{(1.4918 - 0.6703)} = 0.4637$$

The expected price using risk-neutral probabilities (P_u, P_d) would suggest a modest IV of 7.69 ($=157.69 - 150.00$), however this is not the expected option payoff since it ignores the fact that the down-state price would have zero IV when the call option would be worthless (out-of-the-money). The asset prices and IV at the up and down states are: $S_{u1} = 223.77$ ($=150.00 \times 1.4918$), $IV_{u1} = 73.77$ ($=223.77 - 150.00$), $S_{d1} = 100.55$, $IV_{d1} = 0$ ($= \max \{0, 100.55 - 150.00\}$). The down state has zero option value (its IV is zero), so that the option value is calculated as:

$$32.54 = \frac{IV_u P_u + IV_d P_d}{1 + R_{fd}} = \frac{73.77 \times 0.4637 + 0.00 \times (1 - 0.4637)}{1 + 5.13\%}$$

The same call option price can be calculated in terms of a replicating portfolio approach. If the seller receives the 32.54 option premium and borrows 57.26 (at the Risk-free rate) to buy a hypothetical 0.5987 fraction of a share for 89.80 ($= 0.5987 \times 150.00$ share price at T_0), its net position in the two up and down

states would be zero: if the option is exercised for 150.00 in the up state, the writer receives the 150.00 exercise price, repays the loan with interest ($60.20 = 57.26 \times 1.0513$) and uses the remaining 89.80 to buy the missing 0.4013 fraction of a share needed to deliver a whole share ($= 0.4013 \times 223.77$); if the option is not exercised in the down state, the writer would sell the stock for 60.20 ($= 0.5987 \times 100.55$) and pay off the loan. The call option value (32.54) equals the cost of the share (89.80) less the loan (57.26), or, in terms of the current share price S_0

$$\text{Call option (32.54)} = S_0 \times \delta - B = 150.00 \times 0.5987 - 57.26$$

Where:

$$\delta \text{ ('Hedge Ratio')} = \frac{IV_u - IV_d}{S_0 (u - d)} = \frac{73.77 - 0.00}{150 (1.4918 - 0.6703)}$$

$$B \text{ Loan} = \frac{(d \times IV_u - u \times IV_d)}{(u - d)} \times \frac{1}{(1 + Rf_d)} = \frac{(0.6703 \times 73.77) - 0}{(1.4918 - 0.6703)} \times \frac{1}{1.0513}$$

Black-Scholes Model

The well-known Black-Scholes Model ('BSM') (a derivation can be found elsewhere) is the continuous time version of the BM, and attempts, like the BM, to estimate future probability weighted expected prices and positive payoffs. It assumes the option can only be exercised at expiry (European Option), unlike the BM which allows American options exercisable at any date.

The BSM is based on the replicating portfolio approach and for a call option can be shown as:

$$\text{Call option} = S_0 \times \delta - B$$

where δ $N(d_1)$
 B $\text{PV of Exercise Price} \times N(d_2)$

Black-Scholes Model (no dividends)

Call option value	=	$S \times N(d_1)$	-	$X e^{-rT} N(d_2)$
Put option value	=	$X e^{-rT} N(-d_2)$	-	$S \times N(-d_1)$

S Current stock price
 X Exercise Price
 T Time to maturity
 r Risk-free rate (continuously compounded)
 e The base of natural logarithms, constant = 2.71828
 $X e^{-rT}$ The amount of cash needed to be invested over a period of time T at a continuously compounded interest rate r in order to receive X at maturity (i.e. it is the PV of the exercise price continuously discounted at the Risk-free rate).

$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(S/X) + rT}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T}$$

$$d_2 = \frac{\ln(S/X) + (r - 0.5\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where σ is the annualised standard deviation of logarithmic stock returns

$N(x)$ The cumulative probability function for a standardised normal variable:

$N(d_1)$ Call option delta (hedge ratio).

$N(d_2)$ Risk-neutral probability of exercise. $X e^{-rT} N(d_2)$ represents the PV of the price paid to acquire the share.

The BSM uses logarithmic returns and defines the relationship between the current price and exercise price in terms of $\ln(S_0 / X)$, adjusting this to estimate the forward value by the Risk-free rate of return.

Using the example for the BM, we can value the option assuming exercising occurs after 1 year. The call option value is 27.0344, lower than the 32.5421 BM figure because the simple one step model is a poor approximation to the continuous stock price distribution assumed in the BSM.

Assumptions		
Current share price (S)		150.0000
Exercise Price (X)		150.0000
Risk-free rate (R_f)	Discrete	5.13 %
Risk-free rate (R_f)	Continuous	5.00 %
Volatility	Standard Deviation	40.00 %
Time to Expiry (T)		1.00 yrs
Dividend yield (q)		-
Option Valuation		
$d_1 = (\ln(150.00 / 150.00) + (5.00\% - 0.00\% + 0.5 \times 40.00\%^2) \times 1.00) / (40.00\% \times \text{SQRT}(1.00))$		0.3250
$d_2 = 0.3250 - 40.00\% \times \text{SQRT}(1)$		-0.0750
$N(d_1) = \text{NORMSDIST}(0.3250)$		0.6274
$N(d_2) = \text{NORMSDIST}(-0.0750)$		0.4701
Share price (150.00) $\times$ $N(d_1)$		94.1114
PV of ex price {150.00 $\times$ $\text{EXP}(-5.00\% \times 1 \text{ years})$ } $\times$ $N(d_2)$		67.0770
Call Price (BSM)		27.0344

The value of the call option will increase as: share price $\uparrow$ (measured by delta), volatility $\uparrow$ (vega), time to expiry $\uparrow$ (theta – change in value as time-to-expiry shortens), Risk-free rate $\uparrow$ (rho), and the lower the fixed exercise price and dividend yield (assumed nil in this example).

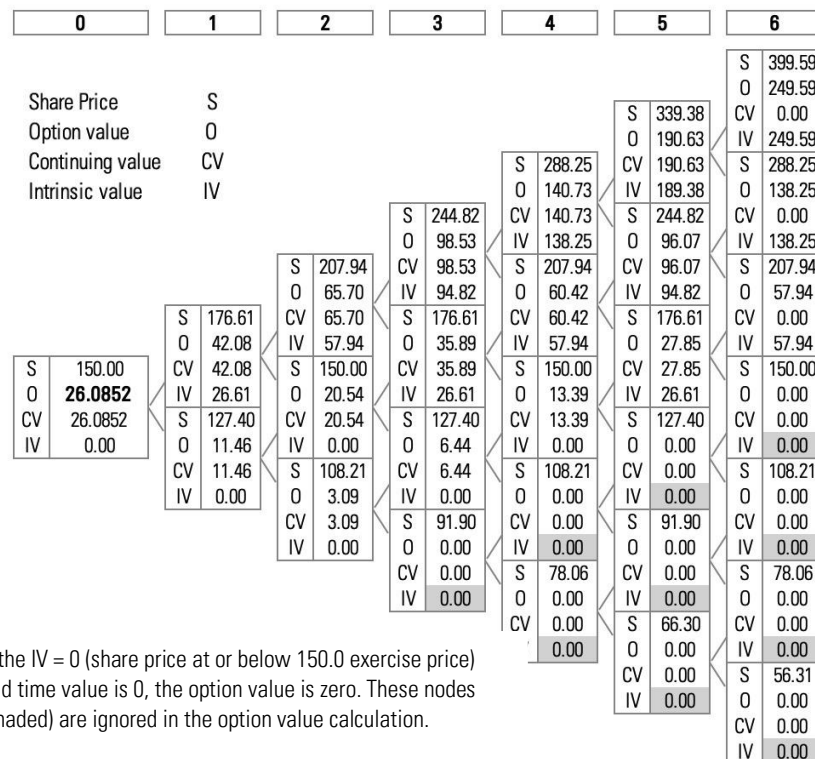
As the number of time periods in the BM increases, the option value converges to the BSM. To equate the BM to the BSM, the time steps in the BM need to be increased significantly, which requires the use of a multi-step tree.

Multi-Step

A 'Binomial Tree' (or 'Lattice') shows expected stock prices, Intrinsic Values and option values for each period until expiry. At the end of each stock price path (a 'node' – for example, the up and down state nodes in the above single period example), the stock price can increase or decrease at the next time period, so that the number of price paths doubles each period (2^n paths after n periods). The tree recombines as $u \times d = 1$, so that there are only $n + 1$ nodes after n periods (trinomial models add a third path between the up and down binomial states). If the option can be exercised at any date before expiry (American option), the value at each node will be the Intrinsic Value at that node or, if higher because of time value, the present value of the risk-neutral expected option value at the next period (the 'Continuing Value'):

$$\text{Continuing Value}_t = \frac{p \times (\text{Option Value}_{t+1u}) + (1 - p) \times (\text{Option Value}_{t+1d})}{1 + \text{Risk-free rate } (R_{fd})}$$

At the final time period t_n (the expiry), the option value will simply be $\max \{0, S_n - X\}$ i.e. the IV or zero as for the one period example above. For a European option, the value at any time-period before expiry will be the Continuing Value, since it cannot be exercised until expiry. Using the same information, but assuming the 1-year period is divided into 6 time steps, the following tree and an option value (26.0852) are shown:



u	1.1774	U = EXP(40.0%*SQRT (1 / 6))	S	= S _{t-1} x U	288.25	= 244.82 x 1.1774
d	0.8493	D = 1/1.1774	O	= MAX (CV, IV)	140.73	= MAX {140.73, 138.25 }
Risk-free return	1.0084	r = EXP(5% x 1/6)	CV	= [pCu + (1-p)Cd] / (1+r)	140.73	= [0.4848 x 190.63 + (1-0.4848) x 96.07] / 1.0084
Risk-neutral probability	0.4848	p = (1.0084 - 0.8493)/(1.1774 - 0.8493)	IV	= Max {Stock Price - Exercise Price, 0}	138.25	= 288.25 - 150.00

Example of workings - cell U⁴

A European Call Option on a non-dividend paying stock should have the same value as its American counterpart, as it would never be optimal to exercise the latter early (this is not the case if dividends are paid, as discussed below). For illustrative and modelling purposes, the above tree is normally presented in the following layout:

European & American Option
(no dividend 26.0852)

	0	1	2	3	4	5	6
S	150.00	176.61	207.94	244.82	288.25	339.38	399.59
IV	0.00	26.61	57.94	94.82	138.25	189.38	249.59
TV	26.09	15.47	7.77	3.70	2.48	1.24	0.00
O	26.0852	42.08	65.70	98.53	140.73	190.63	249.59
S	127.40	150.00	176.61	207.94	244.82	288.25	
IV	0.00	0.00	26.61	57.94	94.82	138.25	
TV	11.46	20.54	9.28	2.48	1.24	0.00	
O	11.46	20.54	35.89	60.42	96.07	138.25	
S	108.21	127.40	150.00	176.61	207.94		
IV	0.00	0.00	0.00	26.61	57.94		
TV	3.09	6.44	13.39	1.24	0.00		
O	3.09	6.44	13.39	27.85	57.94		
S	91.90	108.21	127.40	150.00			
IV	0.00	0.00	0.00	0.00			
TV	0.00	0.00	0.00	0.00			
O	0.00	0.00	0.00	0.00			
S	78.06	91.90	108.21				
IV	0.00	0.00	0.00				
TV	0.00	0.00	0.00				
O	0.00	0.00	0.00				
S	66.30	78.06					
IV	0.00	0.00					
TV	0.00	0.00					
O	0.00	0.00					
S	56.31						
IV	0.00						
TV	0.00						
O	0.00						

0.00 Option worthless

Model assumptions	
Share price today	150.00
ExercisePrice	150.00
Risk Free interest rate	5.00%
Volatility	40.0%
Expiry date	1.00 yrs
Time step (fraction of year)	1/6
Call (c) or Put (p)	c
Total number of time periods	6
Dividend yield per year	0.0%
Dividend yield per period	0.00%

A European option cannot be exercised before expiry, so the option value will reflect its continuing value.

S Share Price
IV Intrinsic Value
TV Time Value
O Option Value

Parameters	
u	$1.177389 = \text{EXP}(40.0\% \times \text{SQRT}(1/6))$
d	$0.849337 = 1/1.1774$
r	$1.008368 = \text{EXP}(5.00\% \times 1/6)$
p	$0.484774 = (\text{EXP}([5.00\% - 0.00\%] \times 1/6) - 0.849337) / (1.177389 - 0.849337)$

BM-BSM Convergence

The call option value calculated above (26.0852) can be calculated using the Binomial distribution, a discrete probability distribution which measures the number of successes (probability 'p') and failures (probability '1-p') in a sequence of independent trials. The call option can be valued as:

$$C_0 = S_0 \times \sum_{n=a}^T \left\{ A \times \left(\frac{u^n d^{T-n}}{(1+r)^T} \right) \right\} - \frac{X}{(1+r)^T} \times \sum_{n=a}^T \left\{ A \right\}$$

PV of expected stock price at each node,
where exercising the option is optimal
(probability weighted).

PV of exercise price at each node
where exercising is optimal
(probability weighted).

Where:

$$A = \frac{T!}{n! (T-n)!} p^n (1-p)^{T-n} = \text{possible combinations} \times \text{probability of each path}$$

Each node can be reached via $T! / [n! (T-n)!]$ possible paths, where T is the number of time steps, r the risk-free rate per step and n the number of up moves.

Node 'n'	p	p^n	$(1-p)^{T-n}$	$P_u \times P_d$	Paths	Binomial probability	In-the-money?	Prob of exercise	State Price	Yr 6 price (S_T) Yr 0 price (S_0)	PV of Expected value of S_T / S_0
A	B	C = B^A	D	E = $C \times D$	F	G		A*	A^* / e^{rt}	$u^n \times d^{(T-n)}$ B	$A^* \times B / e^{rt}$
6	0.4848	0.0130	1.0000	0.0130	1	0.0130	✓	0.0130	0.0123	2.6639	0.0329
5	0.4848	0.0268	0.5152	0.0138	6	0.0828	✓	0.0828	0.0787	1.9217	0.1513
4	0.4848	0.0552	0.2655	0.0147	15	0.2199	✓	0.2199	0.2092	1.3862	0.2900
3	0.4848	0.1139	0.1368	0.0156	20	0.3116	✗				
2	0.4848	0.2350	0.0705	0.0166	15	0.2484	✗				
1	0.4848	0.4848	0.0363	0.0176	6	0.1056	✗				
0	0.4848	1.0000	0.0187	0.0187	1	0.0187	✗				
1.0000								0.3157			0.4742

$$\text{Node 4} = (0.4848^4) \times \{(1 - 0.4848)^{(6-4)}\} = 0.0147$$

Exercise price	150.00	Share Price	150.00
Discount factor	x 0.9512		
Prob. of exercise	x 0.3157	Payoff factor	0.4742
	45.0389		71.1240
Value 26.0852			

	0	1	2	3	4	5	6
	S 150.00	U S 176.61 D S 127.40	UU S 207.94 UD S 150.00 DD S 108.21	UUU S 244.82 UUD S 176.61 UDD S 127.40 DDD S 91.90	UUUU S 288.25 UUUD S 207.94 UUDD S 150.00 UDDD S 108.21 DDDD S 78.06	UUUUU S 339.38 UUUUUD S 244.82 UUUDD S 176.61 UUDDD S 127.40 UUDDDD S 91.90 UUDDDD S 66.30	UUUUUU S 399.59 UUUUUD S 288.25 UUUDD S 207.94 UUUUDD S 150.00 UUDDDD S 108.21 UUDDDD S 78.06 UUDDDD S 56.31
Node 'n'	Share price S _i	Return (S _i / S ₀) - 1	Return ln (S _i / S ₀)				
6	399.59	166.4%	98.0%				
5	288.25	92.2%	65.3%				
4	207.94	38.6%	32.7%				
3	150.00	0.0%	0.0%				
2	108.21	-27.9%	-32.7%				
1	78.06	-48.0%	-65.3%				
0	56.31	-62.5%	-98.0%				

	Discrete	Continuous
Mean	157.69	5.13%
Varia	4214.13	18.7%
σ	64.92	43.3%
	r - 0.5v	-3.000%

The Excel function can also be used:

Up factor	1.1774
Down factor	0.8493
Max up moves at which out-of-the-money	3.0000 = INT {LN [150 [X] / 150 [S ₀] x 0.8493^6] / LN [1.1774 / 0.8493]}
Total trials	6 Time steps
Risk neutral probability	0.4848
Risk free return per time step	1.0084
Probability of success	0.5660 = 0.4848 x (1.1774 / 1.0084)
Current share price factor	0.4742 = 1-BINOMDIST(3 , 6 , 0.5660, TRUE)
Exercise price factor	0.3157 = 1-BINOMDIST(3 , 6 , 0.4848, TRUE)
Call Price	26.0852 = 150 x 0.4742 - 150 / 1.0084^(6) x 0.3157

The number of time steps can be increased to check convergence to the BSM, here up to 1000 time steps (the 1-step tree (32.54) overstates the BSM value (27.0344) by about 20%, so many more time steps are required before the binomial value can be relied on):

EUROPEAN CALL OPTION - BINOMIAL - 1, 2, 3, ..., 1000 TIME PERIODS - 1 YEAR (EXTRACTS)									
Time Steps in 1 year	Up	Down	1 + r _{fd}	Risk neutral p	Last excluded up-count	P of success	d	Sm*	Binomial Call
1	1.491825	0.670320	1.0513	0.463724	0	0.658055	0.6581	0.4637	32.54
2	1.326896	0.753638	1.0253	0.473917	1	0.613313	0.3762	0.2246	24.38
3	1.259784	0.793787	1.0168	0.478586	1	0.592949	0.6378	0.4679	28.91
4	1.221403	0.818731	1.0126	0.481403	2	0.580683	0.4421	0.2851	25.63
5	1.195884	0.836202	1.0101	0.483339	2	0.572266	0.6336	0.4688	28.16
7	1.163213	0.859688	1.0072	0.485893	3	0.561174	0.6318	0.4692	27.83
8	1.151910	0.868123	1.0063	0.486796	4	0.557251	0.4939	0.3348	26.32
9	1.142631	0.875173	1.0056	0.487546	4	0.553998	0.6308	0.4694	27.65
10	1.134839	0.881182	1.0050	0.488180	5	0.551243	0.5076	0.3482	26.46
11	1.128179	0.886384	1.0046	0.488727	5	0.548871	0.6302	0.4695	27.54
12	1.122401	0.890947	1.0042	0.489204	6	0.546800	0.5179	0.3583	26.55
13	1.117328	0.894992	1.0039	0.489626	6	0.544972	0.6298	0.4696	27.46
996	1.012755	0.987405	1.0001	0.498812	498	0.505149	0.6154	0.4575	27.03
997	1.012749	0.987412	1.0001	0.498812	498	0.505146	0.6274	0.4701	27.04
998	1.012742	0.987418	1.0001	0.498813	499	0.505144	0.6154	0.4575	27.03
999	1.012736	0.987424	1.0001	0.498814	499	0.505141	0.6274	0.4701	27.04 *
1000	1.012729	0.987431	1.0001	0.498814	500	0.505139	0.6154	0.4575	27.03

* Workings - time step 999:

Up	=	EXP(40.00% x SQRT(1 / 999))	=	1.012736	Down = 1 / U
1 + r*	=	EXP(5.00% x 1 / 999)	=	1.0001	
p*	=	(1.0001 - 0.9874) / (1.0127 - 0.9874)	=	0.498814	
Min moves	=	INT(LN(150.0 / (150.0 x (0.9874^999))) / LN(1.0127 / 0.	=	499	
Pr success	=	0.4988 x 1.0127 / 1.0001	=	0.505141	
(P)(Px)	=	(1 - BINOMDIST(499, 999, 0.5051, TRUE))	=	0.6274	
P	=	(1 - BINOMDIST(499, 999, 0.4988, TRUE))	=	0.4701	
Binom Call	=	0.6274 x 150.0 - 0.4701 x 142.68 (PV of the ex. price)	=	27.04	

Pricing with Dividends

Dividends reduce the value of a call option since the share price falls by the amount of the dividend when the share goes ex-dividend and the option holder does not receive the cash payout. One method of incorporating dividends into the BM is by assuming they are paid at a continuous yield (q%) and adjusting downwards the risk-neutral probability to $P = (e^{(r-q)t} - d) / (u - d)$. Dividends can make early exercise worthwhile for an American option, which the BSM cannot capture in its basic form. Using the 6 period example, assuming a 6.00% annual dividend yield (1.00% each time step), the option value reduces from 26.0852 to 20.8882 for a European option and 21.5364 for an American option:

European Option
(dividend 20.8882 vs. no dividend 26.0852)

	0	1	2	3	4	5	6
S	150.00	176.61	207.94	244.82	288.25	339.38	399.59
IV	0.00	0.00	0.00	0.00	0.00	0.00	249.59
CV	20.89	35.54	58.29	91.29	135.02	187.25	0.00
O	20.8882	35.54	58.29	91.29	135.02	187.25	249.59
S	127.40	150.00	176.61	207.94	244.82	288.25	
IV	0.00	0.00	0.00	0.00	0.00	138.25	
CV	9.02	17.15	31.72	56.30	93.63	0.00	
O	9.02	17.15	31.72	56.30	93.63	138.25	
S	108.21	127.40	150.00	176.61	207.94		
IV	0.00	0.00	0.00	0.00	57.94		
CV	2.38	5.29	11.75	26.10	0.00		
O	2.38	5.29	11.75	26.10	57.94		
S	91.90	108.21	127.40	150.00			
IV	0.00	0.00	0.00	0.00			
CV	0.00	0.00	0.00	0.00			
O	0.00	0.00	0.00	0.00			
S	78.06	91.90	108.21				
IV	0.00	0.00	0.00				
CV	0.00	0.00	0.00				
O	0.00	0.00	0.00				
S	66.30	78.06					
IV	0.00	0.00					
CV	0.00	0.00					
O	0.00	0.00					
S	56.31						
IV	0.00						
CV	0.00						
O	0.00						

A European option cannot be exercised before expiry (the IV is set to zero for modelling purposes)

0.00 Early exercise worthwhile (O = IV)

0.00 Option worthless

American Option
(dividend 21.5364 vs. no dividend 26.0852)

	0	1	2	3	4	5	6
S	150.00	176.61	207.94	244.82	288.25	339.38	399.59
IV	0.00	26.61	57.94	94.82	138.25	189.38	249.59
TV	21.54	10.10	2.41	0.00	0.00	0.00	0.00
O	21.5364	36.71	60.35	94.82	138.25	189.38	249.59
S	127.40	150.00	176.61	207.94	244.82	288.25	
IV	0.00	0.00	26.61	57.94	94.82	138.25	
TV	9.24	17.60	5.97	0.00	0.00	0.00	
O	9.24	17.60	32.58	57.94	94.82	138.25	
S	108.21	127.40	150.00	176.61	207.94		
IV	0.00	0.00	0.00	26.61	57.94		
TV	2.43	5.40	11.98	0.00	0.00		
O	2.43	5.40	11.98	26.61	57.94		
S	91.90	108.21	127.40	150.00			
IV	0.00	0.00	0.00	0.00			
TV	0.00	0.00	0.00	0.00			
O	0.00	0.00	0.00	0.00			
S	78.06	91.90	108.21				
IV	0.00	0.00	0.00				
TV	0.00	0.00	0.00				
O	0.00	0.00	0.00				
S	66.30	78.06					
IV	0.00	0.00					
TV	0.00	0.00					
O	0.00	0.00					
S	56.31						
IV	0.00						
TV	0.00						
O	0.00						

This simple 6-step model should not be compared to the more accurate BSM value below, but is used to illustrate the effect of dividends

Model assumptions	
Share price today	150.00
ExercisePrice	150.00
Risk Free interest rate	5.00%
Volatility	40.0%
Expiry date	1.00 yrs
Time step (fraction of year)	1/6
Call (c) or Put (p)	c
Total number of time periods	6
Dividend yield per year	6.0%
Dividend yield per period	1.00%

Parameters	
u	1.177389 = EXP(40.0%*SQRT(1 / 6))
d	0.849337 = 1/1.1774
r	1.008368 = EXP (5.00% x 1/6)
p	0.454189 = (EXP ([5.00% - 6.00%] x 1/6) - 0.849337) / (1.177389 - 0.849337)

The BSM incorporates a continuous dividend yield as follows:

Assumptions	
Current share price (S)	150.0000
Exercise Price (X)	150.0000
Risk-free rate (R _{fd})	Discrete 5.13 %
Risk-free rate (R _{fc})	Continuous 5.00 %
Volatility	Standard Deviation 40.00 %
Time to Expiry (T)	1.00 yrs
Dividend yield (q)	6.00 %
Option Valuation	
d1 = (LN(150.00 / 150.00) + (5.00% - 6.00% + 0.5 x 40.00%^2) x 1.00) / (40.00% x SQRT(1.00))	0.1750
d2 = 0.1750 - 40.00% x SQRT(1)	-0.2250
N(d1) = NORMSDIST (0.1750)	0.5695
N(d2) = NORMSDIST (-0.2250)	0.4110
Share price (141.26) x N(d1)	80.4446
PV of ex price { 150.00 x EXP(-5.00% x 1 years) } x N(d2)	58.6418
Call Price (BSM)	21.8028

Monte Carlo Simulation

The BSM assumes the stock price follows a 'stochastic' process (geometric Brownian motion) and grows at its expected rate of return plus a random element determined by its volatility. Because the option can be replicated by a hedged position in the stock and borrowing, it can be valued as if investors are risk-neutral, with the stock price growing at the risk-free rate ($r - q$ where a continuous dividend yield is paid) and the resulting payoff discounted at the risk-free rate.

The stock price at a future date can be simulated as:

$$S_t = S_0 e^{[(r - q - 0.5\sigma^2)t + \sigma\sqrt{t}Z]}$$

Where:

S_t, S_0	Share price at time t and 0
r	Risk-free rate (continuous)
q	Continuous dividend yield
σ	Volatility
t	Time from the valuation date (time to expiry for a European option)
Z	Random variable that follows a standard normal distribution (= NORMSINV(RAND())) using Excel

The $-0.5\sigma^2$ term adjusts the expected return to the expected logarithmic return (see the example on p.14, where the discrete mean return is 5.13% and the continuous mean is -2.98%, which converges to -3.00% (= 5.00% continuous return $- 0.5 \times 40\%^2$)).

The share price equation can be run over as many simulated price paths as required, and for each predicted stock price the payoff can be calculated as for a European Option in the Binomial Tree (=MAX {share price – exercise price, 0}). The average payoff discounted at the risk-free rate gives an estimate of the call price, which converges on the BSM value (27.0344) as the number of simulated paths increases.